1. (10 marks) *True or false?* / @

Circle 'T' if the statement is true.
Circle 'F' if the statement is false.
For this question, you should assume that φ and ψ are WFFs of SL.

T F If ψ is inconsistent, then φ is not a tautology.

T F If φ is inconsistent then φ does not entail φ.

T F "Kant believed that" is a truth functional connective.

T F "$\exists x(Fx \rightarrow Gx)$" is a valid MPL WFF.

T F If X is an inconsistent set of MPL WFFs, then no member of X entails a member of X.

T F If φ entails ψ then φ is consistent.

T F If φ is a tautological conditional, then the consequent of φ is not inconsistent.

T F If X is an inconsistent set of MPL WFFs, then some member of X is not valid.

T F The following argument can be shown to be valid in SL: "If everyone is hungry, then Slim is not hungry. Everyone is hungry. So, Slim is not hungry."

T F φ is not a WFF of MPL.

/10

2. (15 marks)

Suppose that a new one-place connective '@', and a new two-place connective '#' are added to SL. You are informed that:

'$((A\#B) \to @A)$' is a tautology

'@A' does not entail '$(A\#B)$'

'$(A\#B)$' does not entail '@B'

'@A' is contingent

'@A' does not entail '$(A \vee A)$'

Fill in the truth tables for '#' and '@':

A	@A
T	F
F	T

1 @

1 bonus pt

A	B	(A#B)
T	T	F
T	F	F
F	T	T
F	F	F

Circle 'T' if the statement is true. Circle 'F' if the statement is false:

2 @

T F '$(@A\&B)$' is contingent.

T F '$@A, (A \to B) \models B$' is a valid sequent.

T F '$(B\#A)$' is logically equivalent to '$(B\&A)$'.

T F '$(@A\#@B)$' does not entail '$(B \vee A)$'.

/15

3. (10 marks)

For each of the following:
Circle "tautology" if it is a WFF of SL that is a tautology.
Circle "contingent" if it is a contingent WFF of SL.
Circle "inconsistent" if it is an inconsistent WFF of SL.
Otherwise, don't circle anything.

$$(A \leftrightarrow (B \rightarrow A))$$

tautology contingent inconsistent

$$((A \vee (B\&C)) \rightarrow (A \vee C))$$

tautology contingent inconsistent

$$((A\& \sim B) \vee (A\&B))$$

tautology contingent inconsistent

$$((A\& \sim A) \vee (B \vee \sim B))$$

tautology contingent inconsistent

$$((A \leftrightarrow C) \rightarrow (A \rightarrow C))$$

tautology contingent inconsistent

$$(B \rightarrow (A \rightarrow B))$$

tautology contingent inconsistent

$$(A \leftrightarrow (C\&(A \vee B)))$$

tautology contingent inconsistent

$$(\sim F \leftrightarrow (A\&A))$$

tautology contingent inconsistent

$$((A \rightarrow B) \vee (B \rightarrow A))$$

tautology contingent inconsistent

$$((A \leftrightarrow B) \rightarrow ((A \vee C) \leftrightarrow (A\&C)))$$

tautology contingent inconsistent

/10

If missing domain, -1
If missing bracket(s), -1

4. (15 marks) 3@

Translate the following statements and arguments into MPL.
Preserve as much structure as possible.
Use the following translation scheme:

a: Aaron
b: Barbara
Fx: x is friendly
Gx: x is greedy

(a) If Aaron and Barbara are both greedy, then someone is friendly.

Domain: All humans.

$$((Ga \& Gb) \rightarrow \exists x Fx)$$

(b) If everyone is greedy, then someone is. Domain = All humans.

$$(\forall x Gx \rightarrow \exists x Gx)$$

(c) Aaron and Barbara are both greedy only if at least one of the two is not friendly.

$$((Ga \& Gb) \rightarrow (\sim Fa \vee \sim Fb))$$

(d) If everyone is greedy then Barbara is greedy. If someone is greedy, then he is friendly. But Aaron is neither greedy nor friendly. So either every greedy person is not greedy or Aaron is friendly. Domain: All humans.

$$(\forall x Gx \rightarrow Gb), \ \forall x(Gx \rightarrow Fx), (\sim Ga \& \sim Fa) \vdash (\forall x(Gx \rightarrow \sim Gx) \vee Fa)$$

(e) Barbara is greedy but not friendly. Aaron is both friendly and greedy. Therefore, not everyone who is greedy is friendly. Domain: All humans

$$(Gb \& \sim Fb), (Fa \& Ga) \vdash \sim \forall x(Gx \rightarrow Fx)$$

/15

5. (10 marks)

Is there an interpretation under which all the following MPL WFFs are true? If yes, then give one such interpretation. If no, explain why there is no such interpretation.

$\sim\exists x(Ax\&Bx)$
$\sim\exists x\sim(Ax\rightarrow\sim Bx)$
$\forall x(\sim Bx\vee\sim Cx)$
$\exists xCx$

Any interpretation which says:
No A are B.
No B are C
Some C exists

1 bonus pt
3 pt for each fulfilled condition

If missing bracket(s), -2 /10

6. (10 marks)

Write down 4 MPL WFFs which are each consistent, but where there is no interpretation under which more than 1 of the WFFs is true.

Eg: $(Fa \& Fb)$
$(Fa \& \sim Fb)$
$(\sim Fa \& Fb)$
$(\sim Fa \& \sim Fb)$

3 pt if all are consistent
7 pt if under no int. will more than 1 is true.

/10

If missing bracket(s), -2

7. (10 marks)

Give a consistent MPL WFF which is false under every interpretation which contains less than 4 elements in its domain.

Eg. $(((\exists x(Ex \& Fx) \& \exists x(Ex \& \sim Fx)) \& \exists x(\sim Ex \& Fx)) \& \exists x(\sim Ex \& \sim Fx))$

/10

8. (10 marks) 5pt

Give an interpretation under which "$\forall x(Hx \& Gx)$" is false and "$\forall x(Hx \rightarrow Gx)$" is true.

5pt Eg: Domain: All animals

Hx: x is a human.

Gx: x is a mammel.

/10

9. (10 marks) 2 @

Give an MPL WFF that is logically equivalent to each of the following WFFs. Your answer must include an existential quantifier if the original WFF contains a universal quantifier, and vice versa.

(MPL WFF φ is logically equivalent to MPL WFF ψ if and only if φ entails ψ and ψ entails φ.)

(a) $\sim\forall x(Hx \vee Gx)$

$\exists x \sim (Hx \vee Gx)$ or $\exists x (\sim Hx \& \sim Gx)$ or $\exists x \sim(\sim Hx \rightarrow Gx)$

(b) $\sim\exists x(\sim Hx \& \sim Gx)$

$\forall x \sim(\sim Hx \& \sim Gx)$ or $\forall x (Hx \vee Gx)$ or $\forall x (\sim Hx \rightarrow Gx)$

(c) $\forall x(Hx \leftrightarrow \sim Gx)$

$\sim\exists x \sim (Hx \leftrightarrow \sim Gx)$ or $\sim\exists x \sim((Hx \& \sim Gx) \vee (\sim Hx \& Gx))$

or $\sim\exists x \sim((Hx \rightarrow \sim Gx) \& (\sim Gx \rightarrow Hx))$

(d) $\exists x(Hx \& Gx)$

$\sim\forall x \sim (Hx \& Gx)$ or $\sim\forall x (\sim Hx \vee \sim Gx)$

or $\sim\forall x (Hx \rightarrow \sim Gx)$

(e) $\exists y(Fy \& \sim Fy)$

Any MPL WFF that is inconsistent.

/10