Problem Set 2

Elementary Logic

Due: 27 November 2006

Name ____

Student ID Number ________________________________

email ____

Mark ________________%

Due **27 November 2006** by **4:00PM**.

Submit your problem set to Ms. Loletta Li in Main Building 302. Make sure your problem set is timestamped. Do not submit assignments by email. Late penalty: 10% for each day late. This problem set will not be accepted after 30 November.

Answer the questions on the problem set itself. Write neatly. If the grader cannot read your handwriting, you will not receive credit.

Be sure that all pages of the assignment are securely stapled together.

Check the course bulletin board for announcements about the assignment.

Do your own work. If you copy your problem set, or permit others to copy, you may fail the course.

1. (10 marks) *True or false?*
   Circle 'T' if the statement is true.
   Circle 'F' if the statement is false.
   For this question, you should assume that φ and ψ are WFFs of SL.

   T F If ψ is inconsistent, then φ is not a tautology.

   T F If φ is inconsistent then φ does not entail φ.

   T F "Kant believed that" is a truth functional connective.

   T F "$\exists x(Fx \to Gx)$" is a valid MPL WFF.

   T F If X is an inconsistent set of MPL WFFs,
   then no member of X entails a member of X.

   T F If φ entails ψ then φ is consistent.

   T F If φ is a tautological conditional, then the consequent of φ is not inconsistent.

   T F If X is an inconsistent set of MPL WFFs, then some member of X is not valid.

   T F The following argument can be shown to be valid in SL: "If everyone is hungry, then Slim is not hungry. Everyone is hungry. So, Slim is not hungry."

   T F φ is not a WFF of MPL.

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2. (15 marks)

Suppose that a new one-place connective '@', and a new two-place connective '#' are added to SL. You are informed that:
'$((A\#B) \rightarrow @A)$' is a tautology
'@A' does not entail '$(A\#B)$'
'$(A\#B)$' does not entail '@B'
'@A' is contingent
'@A' does not entail '$(A \vee A)$'

Fill in the truth tables for '#' and '@':

A	@A
T	
F	

A	B	(A#B)
T	T	
T	F	
F	T	
F	F	

Circle 'T' if the statement is true. Circle 'F' if the statement is false:

T F '$(@A\&B)$' is contingent.

T F '$@A, (A \rightarrow B) \models B$' is a valid sequent.

T F '$(B\#A)$' is logically equivalent to '$(B\&A)$'.

T F '$(@A\#@B)$' does not entail '$(B \vee A)$'.

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3. (10 marks)

For each of the following:
Circle "tautology" if it is a WFF of SL that is a tautology.
Circle "contingent" if it is a contingent WFF of SL.
Circle "inconsistent" if it is an inconsistent WFF of SL.
Otherwise, don't circle anything.

$(A \leftrightarrow (B \rightarrow A))$

tautology contingent inconsistent

$((A \vee (B\&C)) \rightarrow (A \vee C))$

tautology contingent inconsistent

$((A\& \sim B) \vee (A\&B))$

tautology contingent inconsistent

$((A\& \sim A) \vee (B \vee \sim B))$

tautology contingent inconsistent

$((A \leftrightarrow C) \rightarrow (A \rightarrow C))$

tautology contingent inconsistent

$(B \rightarrow (A \rightarrow B))$

tautology contingent inconsistent

$(A \leftrightarrow (C\&(A \vee B)))$

tautology contingent inconsistent

$(\sim F \leftrightarrow (A\&A))$

tautology contingent inconsistent

$((A \rightarrow B) \vee (B \rightarrow A))$

tautology contingent inconsistent

$((A \leftrightarrow B) \rightarrow ((A \vee C) \leftrightarrow (A\&C)))$

tautology contingent inconsistent

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4. (15 marks)

Translate the following statements and arguments into MPL.
Preserve as much structure as possible.
Use the following translation scheme:

a: Aaron
b: Barbara
Fx: x is friendly
Gx: x is greedy

(a) If Aaron and Barbara are both greedy, then someone is friendly.

(b) If everyone is greedy, then someone is.

(c) Aaron and Barbara are both greedy only if at least one of the two is not friendly.

(d) If everyone is greedy then Barbara is greedy. If someone is greedy, then he is friendly. But Aaron is neither greedy nor friendly. So either every greedy person is not greedy or Aaron is friendly.

(e) Barbara is greedy but not friendly. Aaron is both friendly and greedy. Therefore, not everyone who is greedy is friendly.

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5. (10 marks)
   Is there an interpretation under which all the following MPL WFFs are true? If yes, then give one such interpretation. If no, explain why there is no such interpretation.

   $\sim\exists x(Ax\&Bx)$
   $\sim\exists x\sim(Ax\rightarrow\sim Bx)$
   $\forall x(\sim Bx\vee\sim Cx)$
   $\exists xCx$

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6. (10 marks)
   Write down 4 MPL WFFs which are each consistent, but where there is no interpretation under which more than 1 of the WFFs is true.

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7. (10 marks)
   Give a consistent MPL WFF which is false under every interpretation which contains less than 4 elements in its domain.

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8. (10 marks)
Give an interpretation under which "$\forall x(Hx \& Gx)$" is false and "$\forall x(Hx \rightarrow Gx)$" is true.

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9. (10 marks)
Give an MPL WFF that is logically equivalent to each of the following WFFs. Your answer must include an existential quantifier if the original WFF contains a universal quantifier, and vice versa.
(MPL WFF φ is logically equivalent to MPL WFF ψ if and only if φ entails ψ and ψ entails φ.)

(a) $\sim\forall x(Hx \vee Gx)$

(b) $\sim\exists x(\sim Hx \& \sim Gx)$

(c) $\forall x(Hx \leftrightarrow \sim Gx)$

(d) $\exists x(Hx \& Gx)$

(e) $\exists y(Fy \& \sim Fy)$

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