

Problem Set 1
Elementary Logic
Due: 3 October 2007

Name _____

Arthur Chin

Student ID Number _____

email _____

Mark _____%

Due 3 October 2007 by 4:00PM.

Submit your problem set to Ms. Loletta Li in Main Building 312. Make sure your problem set is timestamped. Do not submit assignments by email. Late penalty: 10% for each day late. This problem set will not be accepted after 5 October.

Answer the questions on the problem set itself. Write neatly. If the grader cannot read your handwriting, you will not receive credit.

Be sure that all pages of the assignment are securely stapled together.

Check the course bulletin board for announcements about the assignment.

Do your own work.

If you copy your problem set, or permit others to copy, you may fail the course.

d. Fill in the blank with an SL conditional to make a correct truth table.

A	B	C	$(\neg(A \leftrightarrow B) \rightarrow \neg(A \leftrightarrow C))$
T	T	T	T
T	T	F	T
T	F	T	F
T	F	F	T
F	T	T	T
F	T	F	F
F	F	T	T
F	F	F	F

or any other logically equivalent conditionals.

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4. (18 marks) Translate the following statements into SL WFFs, using the given translation scheme. Preserve as much structure as possible.

- A: Peter is the murderer.
- B: Peter knows the victim.
- C: John knows the victim.
- D: Paul knows the victim.
- E: Peter has the motive to kill.
- F: John has the motive to kill.
- G: Paul has the motive to kill.

a. All that is required for Peter to be the murderer is that he knows the victim.

$$(B \rightarrow A) \text{ or } (B \leftrightarrow A)$$

b. Peter, John and Paul do not all know the victim, but at least two of them do.

$$(\neg(B \& C \& D) \& ((B \& C) \vee (C \& D)) \vee (B \& D))$$

c. John does not have the motive to kill even though he knows the victim.

$$(\neg F \& C) \text{ or } \neg(C \rightarrow F)$$

d. Not only does Paul know the victim, but he also has the motive to kill if Peter is not the murderer.

$$(D \& (\neg A \rightarrow G)) \text{ or } (\neg A \rightarrow (D \& G))$$

e. Neither Peter nor John has the motive to kill unless Peter is the murderer.

$$(\neg(E \vee F) \vee A) \text{ or } ((\neg E \& \neg F) \vee A) \text{ or } (\neg(\neg E \& \neg F) \rightarrow A)$$

f. Peter's having a motive to kill is both necessary and sufficient for his being the murderer, provided that it is false that both John and Paul know the victim.

$$(\neg(C \& D)) \rightarrow (E \leftrightarrow A)$$

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5. (17 marks)

a. Translate the following statements into SL WFFs, using the given translation scheme. Preserve as much structure as possible.

A: Cathy gets an A in her logic course.

B: Cathy gets a B in her logic course.

C: Cathy will be happy.

D: Cathy does all the assignments.

E: Cathy's mother will buy Cathy a car.

1. Cathy will be happy if either she gets an A or a B in her logic course.

$$(A \vee B) \rightarrow C$$

2. Cathy gets a B in her logic course if and only if she does all the assignments.

$$(B \leftrightarrow D)$$

3. It is not true that either Cathy will not be happy or she does all the assignments.

$$\neg (\neg C \vee D) \text{ or } (\neg C \wedge D)$$

4. If Cathy gets an A but not a B in her logic course then her mother will buy her a new car.

$$((A \wedge \neg B) \rightarrow E)$$

b. Given that all the statements 1-4 are true, will Cathy's mother buy her a new car?

can't be determined

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6. (5 marks)

It is not always appropriate to translate an "If ... then ..." statement into an SL WFF using the truth-functional connective " \rightarrow ". Briefly explain why it might not be appropriate to translate the statement "If chickens have lips, then pigs can fly" into " $(C \rightarrow P)$ ". (C: Chickens have lips. P: Pigs can fly.)

The truth of the counterfactual statement depends upon the inferential relationship between 'chickens have lips' and 'pigs can fly'. As even if it is true that chickens have lips it does not follow that pigs can fly, the statement is false. Yet if it is translated into " $(C \rightarrow P)$ ", it will become true since its antecedent is false.

7. (6 marks)

Is "because" a truth-functional connective? Using examples, explain why or why not.

No. For instance, '7 is a prime number', '7 is divisible only by itself and 1', and 'Arthur is a male' are all true statements. Yet, '7 is a prime number because 7 is divisible only by itself and 1' will be true, while '7 is a prime number because Arthur is a male' will be false.

8. (15 marks)

For each of the following:

Circle "tautology" if it is a WFF of SL that is a tautology.

Circle "contingent" if it is a contingent WFF of SL.

Circle "inconsistent" if it is an inconsistent WFF of SL.

Otherwise, don't circle anything.

tautology	$((A \& \sim B) \rightarrow \sim A)$ contingent	inconsistent
tautology	$((A \vee (B \& C)) \rightarrow (A \vee C))$ contingent	inconsistent
tautology	$((A \vee C) \rightarrow (A \vee \sim A))$ contingent	inconsistent
tautology	$(A \leftrightarrow (C \rightarrow A)) \& B$ contingent	inconsistent
tautology	$((\sim A \leftrightarrow \sim B) \& ((\sim B \leftrightarrow A) \& (C \rightarrow C)))$ contingent	inconsistent
tautology	$(C \leftrightarrow ((A \& B) \rightarrow (C \vee A)))$ contingent	inconsistent
tautology	$((A \& B) \leftrightarrow (\sim A \vee \sim B))$ contingent	inconsistent
tautology	$(\sim \sim A \vee (\sim B \rightarrow C))$ contingent	inconsistent
tautology	$((\sim A \rightarrow (B \rightarrow C)) \vee ((B \rightarrow C) \rightarrow A))$ contingent	inconsistent
tautology	$((A \leftrightarrow B) \rightarrow ((A \vee C) \leftrightarrow (A \& C)))$ contingent	inconsistent

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