Problem Set 2

Elementary Logic

Due: 21 November 2007

Name ______________________________

Student ID Number ______________________________

email ______________________________

Mark ____________%

Due **21 November 2007** by **4:00PM**.

Submit your problem set to Ms. Loletta Li in Main Building 302. Make sure your problem set is timestamped. Do not submit assignments by email. Late penalty: 10% for each day late. This problem set will not be accepted after 23 November.

Answer the questions on the problem set itself. Write neatly. If the grader cannot read your handwriting, you will not receive credit.

Be sure that all pages of the assignment are securely stapled together.

Check the course bulletin board for announcements about the assignment.

Do your own work. If you copy your problem set, or permit others to copy, you may fail the course.

1. (15 marks) *True or false?*
   Circle 'T' if the statement is true.
   Circle 'F' if the statement is false.

T F Any inductive argument can be made valid by adding one extra premise.

T F If a set of MPL WFFs is inconsistent and φ is a member of that set,

then φ is inconsistent with every other member within the set.

T F "$\forall x(Gx \rightarrow Gy)$" is a valid MPL WFF.

T F "It is possible that" is a truth functional connective.

T F The following argument can be shown to be valid in MPL: "If someone is

here, we can start. We cannot start. So, someone is not here."

T F If an SL WFF φ is a tautological conjunction, then each conjunct of φ is consistent.

T F There is no interpretation under which " $\forall x(Hx \& Gx)$" is false

and " $\forall x(Hx \rightarrow Gx)$" is true.

T F If X is an inconsistent set of MPL WFFs, then some member of X is inconsistent.

T F If X is a consistent set of MPL WFFs, then every member of X is consistent.

T F "$\exists x(Fx \rightarrow Gx)$" is a valid MPL WFF.

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2. (16 marks)
For each of the following:
Circle "valid" if it is a valid SL sequent.
Circle "invalid" if it is an invalid SL sequent.
Otherwise, don't circle anything.

$$(A \leftrightarrow (B \rightarrow A)) \models A$$

valid invalid

$$(A \rightarrow B), (A \rightarrow C) \models (B \rightarrow C)$$

valid invalid

$$((A \rightarrow B) \rightarrow C), B, {\sim}C \models A$$

valid invalid

$$(A \& \sim A), (B \vee {\sim}B) \models (C \vee {\sim}C)$$

valid invalid

$$({\sim}A \& B) \models {\sim}(A \& B)$$

valid invalid

$$(B \rightarrow (A \rightarrow B)) \models (B \rightarrow A)$$

valid invalid

$$(A \& \sim B) \models (A \vee B)$$

valid invalid

$$((A \rightarrow B) \rightarrow ({\sim}B \rightarrow {\sim}A)) \models C$$

valid invalid

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3. (21 marks)

Translate the following statements and arguments into MPL.
Preserve as much structure as possible.
Use the following translation scheme:

Domain: The set of all human beings.
m: Mary
p: Peter
Cx: x is clever.
Hx: x is happy.
Lx: x is a student in logic class.
Px: x is a professor.

(a) Either Mary is clever or Peter is happy, and Peter is clever if and only if he is a student in the logic class.

(b) If any of the students in the logic class is clever, then no professor will be unhappy.

(c) All students in the logic class who are happy are clever.

(d) No professor will be happy unless not all students in the logic class are happy.

(e) All and only students in the logic class are happy.

(f) Provided that Peter is a student in the logic class, Mary will be happy if and only if she is clever.

(g) If every person is either clever or happy, then Peter is not a student in the logic class. Both Peter are Mary are students in the logic class. Therefore, someone is both not clever and not happy.

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4. (10 marks)

Give an MPL WFF that is logically equivalent to each of the following WFFs. Your answer must include an existential quantifier if the original WFF contains a universal quantifier, and vice versa. (MPL WFF φ is logically equivalent to MPL WFF ψ if and only if φ entails ψ, and ψ entails φ.)

(a) $\sim\exists x(Hx \rightarrow Gx)$

(b) $\sim\forall x(Kx\&Gx)$

(c) $\exists y(Fy\& \sim Fy)$

(d) $\sim\forall x(\sim Hx \vee Gx)$

(e) $\forall x(Hx\& \sim Gx)$

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5. (20 marks)

Determine whether the following sequents are valid. If a sequent is valid, write "valid". If not, give an interpretation which shows that the sequent is not valid.

$\exists x(Px \& Qx), \sim Pa \models \sim Qa$

$\forall x(Px \vee Qx) \models \sim\exists x(Px \& \sim Qx)$

$Pa, \sim\exists x \sim (Px \rightarrow Qx) \models \exists x Qx$

$\forall x Px, \exists x Qx \models \exists x(Px \& Qx)$

$\forall x(Px \rightarrow Qx), \exists x(Qx \rightarrow Rx), Pa \models Ra$

$\forall x(Px \vee Qx) \models (\forall x Px \vee \forall x Qx)$

$(\forall x Px \vee \forall x Qx) \models \forall x(Px \vee Qx)$

$(\forall x(Px \rightarrow Qx) \rightarrow \exists y \sim Ry), \exists x \sim Px \models (\forall x Rx \rightarrow \exists y(Py \& Qy))$

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6. (20 marks)
For each of the following, circle either "Yes" or "No".

Is there an interpretation under which all the following MPL WFFs are true?

$\forall x(Bx \rightarrow Cx)$
$\sim\forall x \sim (Ax\&(Dx\&Cx))$
$\exists y(By\& \sim Cy)$
$\exists x(Ax\&(\sim Cx \vee Bx))$

Yes No

Is there a consistent MPL WFF which is false under every interpretation containing more than 17 elements in its domain?

Yes No

Is there a set of 7 MPL WFFs such that each pair in the set is consistent, but the entire set is inconsistent?

Yes No

Is there an interpretation under which " $\forall x(Kx\&Bx)$" is false and " $\forall x(Kx \rightarrow Bx)$" is true?

Yes No

Is there an SL WFF which contains no sentence letters other "A" and "B", and which entails "$((A\&D) \rightarrow B)$", and which is entailed by "$((C \vee A) \leftrightarrow (B \vee A))$"?

Yes No

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