Problem Set 2

Elementary Logic

Due: 21 November 2007

Name Arthur Chin

Student ID Number ______

email ______

Mark ______%

Due **21 November 2007** by **4:00PM**.

Submit your problem set to Ms. Loletta Li in Main Building 302. Make sure your problem set is timestamped. Do not submit assignments by email. Late penalty: 10% for each day late. This problem set will not be accepted after 23 November.

Answer the questions on the problem set itself. Write neatly. If the grader cannot read your handwriting, you will not receive credit.

Be sure that all pages of the assignment are securely stapled together.

Check the course bulletin board for announcements about the assignment.

Do your own work. If you copy your problem set, or permit others to copy, you may fail the course.

1. (15 marks) *True or false?* 1.5
Circle 'T' if the statement is true.
Circle 'F' if the statement is false.

T F Any inductive argument can be made valid by adding one extra premise.

T F If a set of MPL WFFs is inconsistent and φ is a member of that set, then φ is inconsistent with every other member within the set.

T F "$\forall x(Gx \to Gy)$" is a valid MPL WFF.

T F "It is possible that" is a truth functional connective.

T F The following argument can be shown to be valid in MPL: "If someone is here, we can start. We cannot start. So, someone is not here."

T F If an SL WFF φ is a tautological conjunction, then each conjunct of φ is consistent.

T F There is no interpretation under which "$\forall x(Hx \& Gx)$" is false and "$\forall x(Hx \to Gx)$" is true.

T F If X is an inconsistent set of MPL WFFs, then some member of X is inconsistent.

T F If X is a consistent set of MPL WFFs, then every member of X is consistent.

T F "$\exists x(Fx \to Gx)$" is a valid MPL WFF.

/15

2. (16 marks) 2 pt @

For each of the following:
Circle "valid" if it is a valid SL sequent.
Circle "invalid" if it is an invalid SL sequent.
Otherwise, don't circle anything.

$(A \leftrightarrow (B \rightarrow A)) \models A$

valid invalid

$(A \rightarrow B), (A \rightarrow C) \models (B \rightarrow C)$

valid invalid

$((A \rightarrow B) \rightarrow C), B, {\sim}C \models A$ → inconsistent premises

valid invalid

$(A \& \sim A), (B \vee {\sim}B) \models (C \vee {\sim}C)$

valid invalid

$({\sim}A \& B) \models {\sim}(A \& B)$

valid invalid

$(B \rightarrow (A \rightarrow B)) \models (B \rightarrow A)$

valid invalid

$(A \& \sim B) \models (A \vee B)$

valid invalid

$((A \rightarrow B) \rightarrow ({\sim}B \rightarrow {\sim}A)) \models C$

valid invalid

/16

3. (21 marks)

3pt @ In each part, −1 for any number of missing bracket(s).

Translate the following statements and arguments into MPL.
Preserve as much structure as possible.
Use the following translation scheme:

Domain: The set of all human beings.
m: Mary
p: Peter
Cx: x is clever.
Hx: x is happy.
Lx: x is a student in logic class.
Px: x is a professor.

(a) Either Mary is clever or Peter is happy, and Peter is clever if and only if he is a student in the logic class.

$((Cm \vee Hp) \& (Cp \leftrightarrow Lp))$

(b) If any of the students in the logic class is clever, then no professor will be unhappy.

$(\exists x(Lx \& Cx) \rightarrow \neg \exists x(Px \& \neg Hx))$ or

$(\exists x(Lx \& Cx) \rightarrow \forall x(Px \rightarrow Hx))$

(c) All students in the logic class who are happy are clever.

$\forall x((Lx \& Hx) \rightarrow Cx)$

(d) No professor will be happy unless not all students in the logic class are happy.

$(\neg \exists x(Px \& Hx) \vee \neg \forall x(Lx \rightarrow Hx))$

$(\forall x(Px \rightarrow \neg Hx) \vee \exists x(Lx \& \neg Hx))$

(e) All and only students in the logic class are happy.

$\forall x(Lx \leftrightarrow Hx)$ or

$(\forall x(Lx \rightarrow Hx) \& \forall x(Hx \rightarrow Lx))$

(f) Provided that Peter is a student in the logic class, Mary will be happy if and only if she is clever.

$(Lp \rightarrow (Hm \leftrightarrow Cm))$

(g) If every person is either clever or happy, then Peter is not a student in the logic class. Both Peter are Mary are students in the logic class. Therefore, someone is both not clever and not happy.

$(\forall x(Cx \vee Hx) \rightarrow \sim Lp), (Lp \& Lm) \models \exists x(\sim Cx \& \sim Hx)$

/21

2 pt @

4. (10 marks)

Give an MPL WFF that is logically equivalent to each of the following WFFs. Your answer must include an existential quantifier if the original WFF contains a universal quantifier, and vice versa. (MPL WFF φ is logically equivalent to MPL WFF ψ if and only if φ entails ψ, and ψ entails φ.)

(a) $\sim\exists x(Hx \rightarrow Gx)$

$\forall x \sim(Hx \rightarrow Gx)$ $\quad \forall x \sim(\sim Hx \vee Gx)$ $\quad \forall x\,(Hx \& \sim Gx)$

(b) $\sim\forall x(Kx \& Gx)$

$\exists x \sim(Kx \& Gx)$ $\quad \exists x(\sim Kx \vee \sim Gx)$ $\quad \exists x\,(Kx \rightarrow \sim Gx)$

(c) $\exists y(Fy \& \sim Fy)$

Any inconsistent MPL WFF with a universal quantifier.

(d) $\sim\forall x(\sim Hx \vee Gx)$

$\exists x \sim(\sim Hx \vee Gx)$ $\quad \exists x \sim(Hx \rightarrow Gx)$ $\quad \exists x(Hx \& \sim Gx)$

(e) $\forall x(Hx \& \sim Gx)$

$\sim\exists x \sim(Hx \& \sim Gx)$ $\quad \sim\exists x(\sim Hx \vee Gx)$ $\quad \sim\exists x(Hx \rightarrow Gx)$

T F T F T

/10

5. (20 marks) 2.5 pt@

Determine whether the following sequents are valid. If a sequent is valid, write "valid". If not, give an interpretation which shows that the sequent is not valid.

1 $\exists x(Px \& Qx), \sim Pa \models \sim Qa$

Invalid.

Px : x is a mammal. a: a lizard called 'Arthur'

Qx : x is an animal.

Any interpretation under which
① something is P and Q
② a is not-P, and Q

2 $\forall x(Px \vee Qx) \models \sim\exists x(Px \& \sim Qx)$

Px : x is a male Domain: all human beings.

Qx : x is a female

Any interpretation under which:
① for every x, x is either P or Q.
② Something is P and not Q.

3 $Pa, \sim\exists x \sim (Px \rightarrow Qx) \models \exists x Qx$

Valid

4 $\forall x Px, \exists x Qx \models \exists x(Px \& Qx)$

Valid

5 $\forall x(Px \rightarrow Qx), \exists x(Qx \rightarrow Rx), Pa \models Ra$ Invalid

Px : x is a warmblooded animal. a: a human called 'Arthur'

Qx : x is an animal

Rx : x is a fish

Any interpretation under which
① All P are Q
② Something is both Q and R.
③ a is P
④ a is not R

6 $\forall x(Px \vee Qx) \models (\forall x Px \vee \forall x Qx)$ Invalid

Domain: set of all human beings

Px : x is a male

Qx : x is a female

Any interpretation under which
① For every x, x is either P or Q
② Something is not P
③ Something is not Q

7 $(\forall x Px \vee \forall x Qx) \models \forall x(Px \vee Qx)$

Valid

8 $(\forall x(Px \rightarrow Qx) \rightarrow \exists y \sim Ry), \exists x \sim Px \models (\forall x Rx \rightarrow \exists y(Py \& Qy))$

Domain: $\{a, b\}$

Ext (P) = $\{a\}$

Ext (Q) = ϕ

Ext (R) = $\{a, b\}$

Invalid.

Domain: set of all human beings

Px : x is a male

Qx : x is a female

Rx : x is a human being

Any interpretation under which:
① Everything is R
② nothing is both P and Q
③ Something is not P
④ Some P are not Q

/20

6. (20 marks) 4 pt @

For each of the following, circle either "Yes" or "No".

Is there an interpretation under which all the following MPL WFFs are true?

$\forall x(Bx \rightarrow Cx)$
$\sim\forall x \sim (Ax\&(Dx\&Cx))$
$\exists y(By\& \sim Cy)$
$\exists x(Ax\&(\sim Cx \vee Bx))$

Yes No → 1st and 3rd WFFs are inconsistent.

Is there a consistent MPL WFF which is false under every interpretation containing more than 17 elements in its domain? If there is such a WFF, then it is true under some interpretation containing less than 17 elements. But then you can construct an interpretation by adding 17 elements to the original domain, but not changing anything else. The WFF will remain to be true.

Yes No

Is there a set of 7 MPL WFFs such that each pair in the set is consistent, but the entire set is inconsistent?

eg
Pa $(\sim Pa \vee Qa)$
Qa $(\sim Pa \vee \sim Qa)$
$(Pa \vee \sim Qa)$ Ra
$(Pa \vee Qa)$

Yes No

Is there an interpretation under which "$\forall x(Kx\&Bx)$" is false and "$\forall x(Kx \rightarrow Bx)$" is true?

Yes No

Is there an SL WFF which contains no sentence letters other "A" and "B", and which entails "$((A\&D) \rightarrow B)$", and which is entailed by "$((C \vee A) \leftrightarrow (B \vee A))$"?

Yes No

/20