Midterm Test

Elementary Logic

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Please write clearly.

You have 60 minutes to complete this test.

Mark _____%

a single expression.

1. (20 marks) 2 pts @

True or false?
Circle 'T' if the statement is true.
Circle 'F' if the statement is false.
Assume that φ is a WFF of SL.

(T) F All arguments are either valid or unsound.

T (F) All good arguments are sound arguments.

T (F) φ might not be an expression of SL.

T (F) If φ contains exactly 5 symbols then φ contains a two-place connective.

(T) F The antecedent of "$(P \rightarrow (Q \vee R))$" is "P".

(T) F The scope of "$\vee$" in "$\sim(A \vee B)$" is "$(A \vee B)$".

(T) F Whenever "$(A\&B)$" is true, "$(A \rightarrow B)$" is also true.

T (F) Logic is a science which tells us how people actually reason.

T (F) If a conditional is a tautology then its consequent is a tautology.

T (F) No WFF of SL contains exactly 4 symbols.

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2. *Make a truth table for each of the following WFFs of SL.* (20 marks) 4 pts @

- Only check the truth values of the main connective.
- -2 for each mistake

e. $((R \leftrightarrow \sim P) \vee Q)$

R	↔	~	P	∨	Q
T	F	F	T	T	T
T	F	F	T	F	F
F	T	T	T	T	T
F	F	T	T	F	F
T	T	F	F	T	T
T	F	F	F	F	F
F	F	T	F	T	T
F	T	T	F	F	F

or

((	R	⇔	~	P	)	∨	Q	)
	T	F	F	T		T	T	
	T	F	F	T		F	F	
	F	T	T	T		T	T	
	F	F	T	T		F	F	
	T	T	F	F		T	T	
	T	F	F	F		F	F	
	F	F	T	F		T	T	
	F	T	T	F		F	F	

b. $((P \vee P) \rightarrow \sim\sim Q)$

P	∨	P	→	~	~	Q
T	T	T	T	F	T	T
T	T	T	F	F	T	F
F	F	F	T	T	F	T
F	F	F	T	T	F	F

c. $((P \& Q) \rightarrow (A \vee R))$

d. $((P \leftrightarrow R) \rightarrow (C \vee Q))$

e. $((\sim\sim P \leftrightarrow \sim Q) \vee \sim R)$

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3. (16 marks) 4 pts @

Fill in the blanks with an SL WFF to make correct truth tables.
Each WFF you write must be a conjunction and must contain at least THREE connectives.

a.

A	B	$(\sim(A \& B) \& \sim(A \& B))$
T	T	F
T	F	T
F	T	T
F	F	T

or any other logically equivalent WFF that fits the requirement.

- If it's not a conjunction, -2
If it doesn't contain at least 3 connectives, -2

b.

A	B	C	$((\sim((A \& B) \& C) \& \sim((A \& \sim B) \& \sim C)) \& \sim((\sim A \& B) \& C))$
T	T	T	F
T	T	F	T
T	F	T	T
T	F	F	F
F	T	T	F
F	T	F	T
F	F	T	T
F	F	F	T

c.

A	B	$(\sim(A \& \sim B) \& \sim(\sim A \& \sim B))$
T	T	T
T	F	F
F	T	T
F	F	F

d.

A	B	C	$((\sim((A \& B) \& C) \& \sim((A \& \sim B) \& C)) \& \sim((\sim A \& B) \& C))$
T	T	T	F
T	T	F	T
T	F	T	F
T	F	F	T
F	T	T	F
F	T	F	T
F	F	T	T
F	F	F	T

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4. (12 marks) *Suppose that "#" is a new connective added to SL.*
You are told that "((A#B)#B)" is a tautology, but "(A#B)" is not a tautology.
If possible, complete the following truth table for "(A#B)".
If it is not possible to complete the truth table, explain why.

No' - 6 pts
Explanation: -6 pts

A	B	$(A\#B)$
T	T	
T	F	
F	T	
F	F	

No. Because if '((A#B)#B)' is a tautology, then '(A#B)' must be a tautology.
~~Hence~~ It is logically impossible for '((A#B)#B)' to be a tautological and '(A#B)' not to be tautological at the same time.
(Pls let me know if you don't understand.)

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5. (20 marks) 2 pts @

For each of the following:
Circle "tautology" if it is a WFF of SL that is a tautology.
Circle "contingent" if it is a contingent WFF of SL.
Circle "inconsistent" if it is an inconsistent WFF of SL.
Otherwise, don't circle anything.

	$(A \rightarrow (A \rightarrow B))$	
tautology	contingent	inconsistent
	$((A \leftrightarrow B) \rightarrow ((A \vee B) \leftrightarrow (A\&B)))$	
tautology	contingent	inconsistent
	$\sim(\sim A \rightarrow (A \leftrightarrow A))$	
tautology	contingent	inconsistent
	$((A \vee B) \vee (\sim A\&B))$	
tautology	contingent	inconsistent
	$(C \leftrightarrow (\sim A \vee (C\&B)))$	
tautology	contingent	inconsistent
	$((A \rightarrow B) \vee (B \rightarrow A))$	
tautology	contingent	inconsistent
	$(A \leftrightarrow (A\&(A \vee B))$	
tautology	contingent	inconsistent → not a WFF
	$(A \leftrightarrow (A\&A))$	
tautology	contingent	inconsistent
	$\sim\sim(\sim\sim A\&\sim A)$	
tautology	contingent	inconsistent
	$(((A \rightarrow B) \rightarrow B) \rightarrow ((A \rightarrow B) \rightarrow A))$	
tautology	contingent	inconsistent

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6. (12 marks)

Assume that each of the following four statements is **false**:

If Andy does not play, then Andy and Stewart remember the song.

Sting does not sing.

Either Andy plays and he remembers the song, or Andy does not play and Stewart does not remember the song.

Sting sings only if Stewart does not remember the song.

(6 pts) 1. *Translate each of the four statements into SL, preserving as much structure as possible. Be sure to write down your translation scheme.*

P : Andy plays. Q : Andy remembers the song.
R : Steward remembers the song. S : Sting sings.

$(\neg P \rightarrow (Q \veebar R))$

$\neg S$

$((P \& Q) \vee (\neg P \& \neg R))$

$(S \rightarrow \neg R)$

6 pts 2. *Does Andy play?*

No.

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Only check truth values under main connective.
- -2 for each mistake.

3 in natural order or in alphabetical order.

c)

p	q	a	r	((p&q)→ (aVr))
T	T	T	T	T
T	T	T	F	T
T	T	F	T	T
T	T	F	F	F
T	F	T	T	T
T	F	T	F	T
T	F	F	T	T
T	F	F	F	T
F	T	T	T	T
F	T	T	F	T
F	T	F	T	T
F	T	F	F	T
F	F	T	T	T
F	F	T	F	T
F	F	F	T	T
F	F	F	F	T

a	p	q	r	((p& q)→ (aVr))
T	T	T	T	T
T	T	T	F	T
T	T	F	T	T
T	T	F	F	T
T	F	T	T	T
T	F	T	F	T
T	F	F	T	T
T	F	F	F	T
F	T	T	T	T
F	T	T	F	F
F	T	F	T	T
F	T	F	F	T
F	F	T	T	T
F	F	T	F	T
F	F	F	T	T
F	F	F	F	T

d)

p	r	c	q	((p↔r)→ (cVq))
T	T	T	T	T
T	T	T	F	T
T	T	F	T	T
T	T	F	F	F
T	F	T	T	T
T	F	T	F	T
T	F	F	T	T
T	F	F	F	T
F	T	T	T	T
F	T	T	F	T
F	T	F	T	T
F	T	F	F	T
F	F	T	T	T
F	F	T	F	T
F	F	F	T	T
F	F	F	F	F

c	p	q	r	((p↔r)→ (cVq))
T	T	T	T	T
T	T	T	F	T
T	T	F	T	T
T	T	F	F	T
T	F	T	T	T
T	F	T	F	T
T	F	F	T	T
T	F	F	F	T
F	T	T	T	T
F	T	T	F	T
F	T	F	T	F
F	T	F	F	T
F	F	T	T	T
F	F	T	F	T
F	F	F	T	T
F	F	F	F	F

e)

p	q	r	((¬¬p↔q)V¬r)
T	T	T	T
T	T	F	T
T	F	T	F
T	F	F	T
F	T	T	F
F	T	F	T
F	F	T	T
F	F	F	T