

Problem Set 1
Elementary Logic
Due: 20 February 2008

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Due 20 February 2007 by 4:00PM.

Submit your problem set to Ms. Loletta Li in Main Building 312. Make sure your problem set is timestamped. Do not submit assignments by email. Late penalty: 10% for each day late. This problem set will not be accepted after 28 February.

Answer the questions on the problem set itself. Write neatly. If the grader cannot read your handwriting, you will not receive credit.

Be sure that all pages of the assignment are securely stapled together.

Check the course bulletin board for announcements about the assignment.

Do your own work.

If you copy your problem set, or permit others to copy, you may fail the course.

1. (15 marks)

True or false?

Circle 'T' if the statement is true.

Circle 'F' if the statement is false.

For this question, you should assume that φ is a WFF of SL.

- T F Every sound argument is a good argument.
- T F There is an SL WFF with truth-value both T and F.
- T F There is an expression of SL containing exactly 14 symbols.
- T F The premises and conclusion of a valid argument can all be false.
- T F No argument with a false conclusion contains a hidden assumption.
- T F If φ contains the symbol "(" then φ contains a one-place connective.
- T F φ is not an expression of SL.
- T F The main connective of " $\sim \sim (\sim C \& B)$ " is "&".
- T F Logic describes how people reason.
- T F A lexically ambiguous word has more than one meaning in a language.
- T F No valid argument has a false conclusion.
- T F Whenever " $(A \rightarrow B)$ " is true, " $(A \& B)$ " is also true.
- T F Some good arguments are not valid arguments.
- T F "The pines of Rome" is a statement.
- T F The premises and conclusion of an invalid argument can all be true.

2. (5 marks)

Which of the following is a valid argument?

Circle "Yes" if it is a valid argument.

Circle "No" if it is not a valid argument.

Yes No If Henry is cold, then Marge is cold.
 If Mary is cold, then Nero is cold.
 If Nero is cold, then Arnie is cold.
 So, if Henry is cold, then Arnie is cold.

Yes No If you feel thirsty, then drink tea.
 You feel thirsty.
 So, drink tea.

Yes No Not all birds lay eggs.
 So, some birds lay eggs.

Yes No If it snows in Hong Kong, then it snows in Hong Kong.

Yes No Tokyo is in Japan.
 Tokyo is not in Japan.
 Nothing is both in Japan and not in Japan.
 Therefore, London is in France.

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3. (15 marks)

Make a correct truth table for each of the following WFFs of SL.

a. $((A \vee B) \leftrightarrow (A \rightarrow B))$

T	T	T	T	T	T	T
T	T	F	F	T	F	F
F	T	T	T	F	T	T
F	F	F	F	F	T	F

b. $((A \& B) \rightarrow \sim C)$

T	T	T	F	F	T
T	T	T	T	T	F
T	F	F	T	F	T
T	F	F	T	T	F
F	F	T	T	F	T
F	F	T	T	T	F
F	F	F	T	F	T
F	F	F	T	T	F

c. $((A \rightarrow C) \leftrightarrow A)$

T	T	T	T
T	F	F	F
F	T	T	T
F	F	F	F

d. $((A \rightarrow \sim C) \vee C)$

T	F	F	T
T	T	F	T
F	T	F	T
F	T	F	T

e. $((A \& \sim C) \& \sim A)$

T	F	F	T
T	T	F	F
F	F	F	T
F	F	T	F

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4. (15 marks)

Fill in the blanks with an SL WFF to make correct truth tables.

a.

B	C	$\sim(C \rightarrow B)$
T	T	F
T	F	F
F	T	T
F	F	F

b.

A	B	C	$((A \leftrightarrow B) \leftrightarrow C) \vee (A \leftrightarrow B) \leftrightarrow C) \vee ((\neg A \leftrightarrow B) \leftrightarrow C) \vee ((\neg A \leftrightarrow \neg B) \leftrightarrow C)$
T	T	T	T
T	T	F	F
T	F	T	T
T	F	F	F
F	T	T	F
F	T	F	T
F	F	T	T
F	F	F	F

c.

A	B	C	$((A \leftrightarrow \neg B) \leftrightarrow C) \vee ((\neg A \leftrightarrow B) \leftrightarrow C)$
T	T	T	F
T	T	F	F
T	F	T	T
T	F	F	F
F	T	T	F
F	T	F	T
F	F	T	F
F	F	F	F

d.

C	$(C \leftrightarrow C)$
T	F
F	F

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5. (5 marks)

Is "because" a truth-functional connective? Using examples, explain why or why not.

No. For instance:

'3 is a prime no.' because 3 is divisible only by itself and 1' is true,
 while '3 is a prime no. because 3 is an odd number' is false,
 even though '3 is a prime no.', '3 is divisible only by itself and 1'
 and '3 is an odd number' are all true.

(* All students have been given

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5 points for this question regardless of their answers.

This is because the concept of 'truth functionality' is discussed in [SL09],
 and materials for [SL09] are not intended to be included in this problem set.)

6. (10 marks)

Translate the following statements into SL.
 Preserve as much structure as possible.
 Use the following translation scheme:

- A: David is early.
- B: Mathilda is at home.
- C: Larry is nice.
- D: Lena is hungry.

(a) If Lena is hungry, then Larry is nice if David is early.

$$(D \rightarrow (A \rightarrow C))$$

(b) Whenever Lena is hungry and David is early, Larry is not nice.

$$((D \& A) \rightarrow \sim C)$$

(c) Mathilda is not at home, whether or not Larry is nice.

$$((C \rightarrow \sim B) \& (\sim C \rightarrow \sim B)) \quad ((C \vee \sim C) \rightarrow \sim B)$$

(d) Lena is hungry only if either Larry is nice or Mathilda is at home.

$$(D \rightarrow (C \vee B))$$

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7. (10 marks)

Assume that each of the following three statements is true:

If he likes chocolate, then he both thinks too much and eats too much.

Either he likes chocolate and he thinks too much, or he does not like chocolate and he thinks too much and eats too much.

He thinks too much.

1. Translate each of the three statements into SL, preserving as much structure as possible. Be sure to write down your translation scheme.

$$(L \rightarrow (T \& E))$$

$$((L \& T) \vee (\sim L \& T \& E))$$

T

L: He likes choc
 T: He thinks too much
 E: He eats too much

2. Does he like chocolate?

$(L \rightarrow (T \& E))$	$((L \& T) \vee (\sim L \& (T \& E)))$
T T TTT	TTT T FTFTTT
FT T	FFT T FT T

6 Cannot be determined /10

8. (15 marks)

For each of the following:

Circle "tautology" if it is a WFF of SL that is a tautology.

Circle "contingent" if it is a contingent WFF of SL.

Circle "inconsistent" if it is an inconsistent WFF of SL.

Otherwise, don't circle anything.

tautology	$((A \vee \sim B) \rightarrow \sim A)$ contingent	inconsistent
tautology	$((A \& (B \& C)) \rightarrow (A \vee C))$ contingent	inconsistent
tautology	$((A \& C) \rightarrow (A \vee \sim A))$ contingent	inconsistent
tautology	$(A \leftrightarrow (C \leftrightarrow A)) \& B$ contingent	inconsistent
tautology	$((\sim A \rightarrow \sim B) \& ((\sim B \leftrightarrow A) \& (C \rightarrow C)))$ contingent	inconsistent
tautology	$(C \rightarrow ((A \& B) \rightarrow (C \vee A)))$ contingent	inconsistent
tautology	$((A \vee B) \leftrightarrow (\sim A \vee \sim B))$ contingent	inconsistent
tautology	$(\sim \sim A \& (\sim B \rightarrow C))$ contingent	inconsistent
tautology	$((\sim A \rightarrow (B \rightarrow C)) \vee ((B \rightarrow C) \rightarrow A))$ contingent	inconsistent
tautology	$((A \leftrightarrow B) \leftrightarrow ((A \vee C) \leftrightarrow (A \& C)))$ contingent	inconsistent

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9. (10 marks)

Assume that each of the following four statements is false:

If Andy does not play, then Andy and Stewart remember the song.

Sting does not sing.

Either Andy plays and he remembers the song, or Andy does not play and Stewart does not remember the song.

Sting sings only if Stewart does not remember the song.

1. Translate each of the four statements into SL, preserving as much structure as possible. Be sure to write down your translation scheme.

$$(\sim P \rightarrow (A \& R)) \quad \sim S$$

$$((P \& A) \vee (\sim P \& \sim R))$$

$$(S \rightarrow \sim R)$$

P: Andy plays

A: Andy remembers song

R: Stewart remembers song

S: Sting sings

// false

$(\sim P \rightarrow (A \& R))$	$\sim S$	$((P \& A) \vee (\sim P \& \sim R))$	$(S \rightarrow \sim R)$
T F F	F F T	F F F F T F F T	T F F T

2. Does Andy play?

No

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