Problem Set 2

Elementary Logic

Due: 15 April 2008

Name Answer key

Student ID Number ______

email ______

Mark ______%

Due **18 April 2008** by **4:00PM.**

Submit your problem set to Ms. Loletta Li in Main Building 312. Make sure your problem set is timestamped. Do not submit assignments by email. Late penalty: 10% for each day late. This problem set will not be accepted after 18 April.

Answer the questions on the problem set itself. Write neatly. If the grader cannot read your handwriting, you will not receive credit.

Be sure that all pages of the assignment are securely stapled together.

Check the course bulletin board for announcements about the assignment.

Do your own work. If you copy your problem set, or permit others to copy, you may fail the course.

1. (15 marks) *True or false?* 1.5e
Circle 'T' if the statement is true.
Circle 'F' if the statement is false.

T F If ψ is consistent, then ψ is not a tautology.

T F "It is true that" is a truth functional connective.

T F If X is an inconsistent set of MPL WFFs,

then some member of X entails a member of X.

T F If φ is inconsistent then φ entails φ.

T F "$\exists x(Gx \rightarrow Gy)$" is not a valid MPL WFF.

T F If φ is a tautological conditional, then the antecedent of φ is inconsistent.

T F If φ is an inconsistent disjunction, then each disjunct of φ is inconsistent.

T F If X is an inconsistent set of MPL WFFs, then some member of X is inconsistent.

T F The following argument can be shown to be valid in SL: "If someone is

not here, then Dave is not here. Someone is not here. So, Dave is not here."

T F If φ does not entail ψ then ψ is consistent.

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2. (~~16~~ 14 marks) 2pv@

For each of the following:
Circle "valid" if it is a valid SL sequent.
Circle "invalid" if it is an invalid SL sequent.
Otherwise, don't circle anything.

$(A \& \sim A), (B \vee \sim B) \models (C \& \sim C)$

valid invalid

T F F F T T F F

$(B \to (A \leftrightarrow B)) \models (B \to A)$

valid invalid

T T T F T F F

$(A \& \sim B) \models (A \& B)$

valid invalid

T F T T F F T T

$(\sim A \& B) \models \sim(A \vee B)$

valid invalid

$((A \to B) \leftrightarrow (\sim B \to \sim A)) \models C$

valid invalid

F T T F T F T F F

$(A \to B), (A \leftrightarrow C) \models (B \to C)$

valid invalid

F T T F F

$(A \to (B \to A)) \models A$

valid invalid

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3. (21 marks)

(a) *Translate the following statements and arguments into MPL. Preserve as much structure as possible. Use the following translation scheme:*
Domain: All human beings
Jx: x is jealous
Cx: x is clever
Ax: x gets an A in logic class
Hx: x is handsome
p: Peter
j: John

2pt@

Peter's not being jealous is both a necessary and sufficient condition for his getting an A in logic class.

$(\neg Jp \leftrightarrow Ap)$

Peter is not jealous unless John is either handsome or clever.

$(\neg Jp \vee (Hj \vee Cj))$ or $(\neg(Hj \vee Cj) \rightarrow \neg Jp)$

Peter is both handsome and jealous, but John is not jealous.

$((Hp \,\&\, Jp) \,\&\, \neg Jj)$

Someone who gets an A in logic class is handsome.

$\forall x (Ax \rightarrow Hx)$ or $\neg \exists x (Ax \,\&\, \neg Hx)$

Everyone is either handsome or jealous but no one is both handsome and not clever.

$(\forall x (Hx \vee Jx) \,\&\, \neg \exists x (Hx \,\&\, \neg Cx))$ or $(\forall x (Hx \vee Jx) \,\&\, \forall x (Hx \rightarrow Cx))$

All handsome people are clever but some clever people are not handsome.

$(\forall x (Hx \rightarrow Cx) \,\&\, \exists x (Cx \,\&\, \neg Hx))$

Someone is clever only if someone is both clever and handsome.

$(\exists x Cx \rightarrow \exists x (Cx \,\&\, Hx))$

4 pt

(b) *Suppose that the statements in (a) are all true. Does Peter get an A in logic class?*

No.

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4. (10 marks) 2 pt @

Give an MPL WFF that is logically equivalent to each of the following WFFs. Your answer must include an existential quantifier if the original WFF contains a universal quantifier, and vice versa. (MPL WFF φ is logically equivalent to MPL WFF ψ if and only if φ entails ψ, and ψ entails φ.)

(a) $\sim\exists x(Fx\&Gx)$

$\forall x \neg(Fx \& Gx)$ or $\forall x(\neg Fx \vee \neg Gx)$

(b) $\sim\forall x(\sim Hx\&Fx)$

$\exists x \neg(\neg Hx \& Fx)$ or $\exists x(Hx \vee \neg Fx)$

(c) $\forall x(Hx \vee \sim Jx)$

$\neg\exists x \neg(Hx \vee \neg Jx)$ or $\neg\exists x(\neg Hx \& Jx)$

(d) $\sim\forall x(Kx \vee Hx)$

$\exists x \neg(Kx \vee Hx)$ or $\exists x(\neg Kx \& \neg Hx)$

(e) $\exists y(By \vee \sim By)$

$\neg\forall y \neg(By \vee \neg By)$

$\neg\forall y(\neg By \& By)$

or any other tautological universally quantified formula.

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5. (20 marks) 2.5 @

Determine whether the following sequents are valid. If a sequent is valid, write "valid". If not, give an interpretation which shows that the sequent is not valid.

$\exists x(Px \vee Qx) \models (\exists xPx \vee \exists xQx)$

Valid

$\exists x(Px \vee Qx), \sim Pa \models \sim Qa$

Domain : human beings
Px : x is a female   a : a male called 'Arthur'   Invalid.
Qx : x is a male

$\forall x(Px \vee Qx) \models \sim\forall x(Px \& \sim Qx)$

Domain : all cats
Px : x is a cat   Invalid.
Qx : x is a dog

$Pa, \sim\exists x(Px \rightarrow Qx) \models \exists xQx$

Domain : all cats   a : cat called 'Arthur' Invalid.
Px : x is a cat
Qx : x is a dog

$(\forall xPx \vee \forall xQx) \models \forall x(Px \vee Qx)$

Valid.

$(\forall x(Px \rightarrow Qx) \rightarrow \exists yRy), \exists xPx \models (\forall xRx \rightarrow \exists y(Py \& \sim Qy))$

Invalid.

Domain : all male human beings
Px : x is a male   Rx : x is a mammal
Qx : x is a human being

$\forall xPx, \forall xQx \models \exists x(Px \& Qx)$

Valid.

$\forall x(Px \rightarrow Qx), \forall x(Qx \rightarrow Rx), Pa \models Ra$

Valid

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6. (20 marks)
For each of the following, circle either "Yes" or "No".

Is there an interpretation under which all the following MPL WFFs are true?

(i) $\exists x(Bx\&Cx)$
$\sim\exists x\sim(Cx\rightarrow(Dx\&Ax))$
(iii) $\forall y(By\rightarrow\sim Cy)$
$\forall x(Ax\rightarrow(\sim Cx\vee Bx))$

(i) and (iii) are inconsistent.

Yes   No

Is there a consistent MPL WFF which is false under every interpretation containing less than 7 elements in its domain? eg.

$$(((((\exists x((Px\&Qx)\&Rx)\&\exists x((Px\&Qx)\&\neg Rx))$$
$$\&\exists x((Px\&\neg Qx)\&\neg Rx))\&\exists x((\neg Px\&Qx)\&Rx))$$
$$\&\exists x((\neg Px\&\neg Qx)\&Rx))\&\exists x((Px\&\neg Qx)\&Rx))$$
$$\&\exists x((\neg Px\&\neg Qx)\&\neg Rx))$$

Yes   No

Is there a set of 7 MPL WFFs such that each pair in the set is inconsistent, but the entire set is consistent?

Yes   No

Is there an interpretation under which "$\forall x(Kx\rightarrow Bx)$" is false and "$\forall x(Kx\vee Bx)$" is true?

Yes   No

Is there an SL WFF which contains no sentence letters other "A" and "B", and which entails "$((A\&D)\rightarrow B)$", and which is entailed by "$(B\&((C\vee A)\leftrightarrow(B\vee A)))$"?

Yes   No   eg $((A\&B)\vee(\neg A\&B))$

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