Problem Set 2

Elementary Logic

Due: 15 April 2008

Name ______________________________

Student ID Number ______________________________

email ______________________________

Mark ______________%

Due **15 April 2008** by **4:00PM**.

Submit your problem set to Ms. Loletta Li in Main Building 312. Make sure your problem set is timestamped. Do not submit assignments by email. Late penalty: 10% for each day late. This problem set will not be accepted after 18 April.

Answer the questions on the problem set itself. Write neatly. If the grader cannot read your handwriting, you will not receive credit.

Be sure that all pages of the assignment are securely stapled together.

Check the course bulletin board for announcements about the assignment.

Do your own work. If you copy your problem set, or permit others to copy, you may fail the course.

1. (15 marks) *True or false?*
Circle 'T' if the statement is true.
Circle 'F' if the statement is false.
φ and ψ are SL WFFs.

T F If ψ is consistent, then ψ is not a tautology.

T F "It is true that" is a truth functional connective.

T F If X is an inconsistent set of MPL WFFs,

then some member of X entails a member of X.

T F If φ is inconsistent then φ entails φ.

T F "$\exists x(Gx \to Gy)$" is not a valid MPL WFF.

T F If φ is a tautological conditional, then the antecedent of φ is inconsistent.

T F If φ is an inconsistent disjunction, then each disjunct of φ is inconsistent.

T F If X is an inconsistent set of MPL WFFs, then some member of X is inconsistent.

T F The following argument can be shown to be valid in SL: "If someone is

not here, then Dave is not here. Someone is not here. So, Dave is not here."

T F If φ does not entail ψ then ψ is consistent.

/15

2. (14 marks)
For each of the following:
Circle "valid" if it is a valid SL sequent.
Circle "invalid" if it is an invalid SL sequent.
Otherwise, don't circle anything.

$$(A\& \sim A), (B \vee \sim B) \models (C\& \sim C)$$

valid invalid

$$(B \rightarrow (A \leftrightarrow B)) \models (B \rightarrow A)$$

valid invalid

$$(A\& \sim B) \models (A\&B)$$

valid invalid

$$(\sim A\&B) \models \sim(A \vee B)$$

valid invalid

$$((A \rightarrow B) \leftrightarrow (\sim B \rightarrow \sim A)) \models C$$

valid invalid

$$(A \rightarrow B), (A \leftrightarrow C) \models (B \rightarrow C)$$

valid invalid

$$(A \rightarrow (B \rightarrow A)) \models A$$

valid invalid

/14

3. (21 marks)

(a) *Translate the following statements and arguments into MPL.*
Preserve as much structure as possible.
Use the following translation scheme:
Domain: All human beings
Jx: x is jealous
Cx: x is clever
Ax: x gets an A in logic class
Hx: x is handsome
p: Peter
j: John

Peter's not being jealous is both a necessary and sufficient condition for his getting an A in logic class.

Peter is not jealous unless John is either handsome or clever.

Peter is both handsome and jealous, but John is not jealous.

Someone who gets an A in logic class is handsome.

Everyone is either handsome or jealous but no one is both handsome and not clever.

All handsome people are clever but some clever people are not handsome.

Someone is clever only if someone is both clever and handsome.

(b) *Suppose that the statements in (a) are all true.*
Does Peter get an A in logic class?

/21

4. (10 marks)

Give an MPL WFF that is logically equivalent to each of the following WFFs. Your answer must include an existential quantifier if the original WFF contains a universal quantifier, and vice versa. (MPL WFF φ is logically equivalent to MPL WFF ψ if and only if φ entails ψ, and ψ entails φ.)

(a) $\sim\exists x(Fx \& Gx)$

(b) $\sim\forall x(\sim Hx \& Fx)$

(c) $\forall x(Hx \vee \sim Jx)$

(d) $\sim\forall x(Kx \vee Hx)$

(e) $\exists y(By \vee \sim By)$

/10

5. (20 marks)

Determine whether the following sequents are valid. If a sequent is valid, write "valid". If not, give an interpretation which shows that the sequent is not valid.

$\exists x(Px \lor Qx) \models (\exists xPx \lor \exists xQx)$

$\exists x(Px \lor Qx), {\sim}Pa \models {\sim}Qa$

$\forall x(Px \lor Qx) \models {\sim}\forall x(Px \& \sim Qx)$

$Pa, {\sim}\exists x(Px \rightarrow Qx) \models \exists xQx$

$(\forall xPx \lor \forall xQx) \models \forall x(Px \lor Qx)$

$(\forall x(Px \rightarrow Qx) \rightarrow \exists yRy), \exists xPx \models (\forall xRx \rightarrow \exists y(Py \& \sim Qy))$

$\forall xPx, \forall xQx \models \exists x(Px \& Qx)$

$\forall x(Px \rightarrow Qx), \forall x(Qx \rightarrow Rx), Pa \models Ra$

/20

6. (20 marks)
For each of the following, circle either "Yes" or "No".

Is there an interpretation under which all the following MPL WFFs are true?

$\exists x(Bx\&Cx)$
$\sim\exists x \sim (Cx \rightarrow (Dx\&Ax))$
$\forall y(By \rightarrow \sim Cy)$
$\forall x(Ax \rightarrow (\sim Cx \vee Bx))$

Yes No

Is there a consistent MPL WFF which is false under every interpretation containing less than 7 elements in its domain?

Yes No

Is there a set of 7 MPL WFFs such that each pair in the set is inconsistent, but the entire set is consistent?

Yes No

Is there an interpretation under which " $\forall x(Kx \rightarrow Bx)$" is false and " $\forall x(Kx \vee Bx)$" is true?

Yes No

Is there an SL WFF which contains no sentence letters other "A" and "B", and which entails "$((A\&D) \rightarrow B)$", and which is entailed by "$(B\&((C \vee A) \leftrightarrow (B \vee A)))$"?

Yes No

/20