

The University of Hong Kong  
Philosophy : PHIL 1006 Elementary Logic  
Final Examination  
20 December 2005  
2:30 p.m. - 3:30 p.m.

Student ID Number \_\_\_\_\_

- Answer all questions.
- You have **one hour** to complete this exam.
- Write your answers on this exam paper.

1. (20 marks)

Circle 'T' if the statement is true. Circle 'F' if the statement is false.

Assume that  $\varphi$  and  $\psi$  are SL WFFs.

- T  F If  $\varphi$  entails  $\psi$  then  $\psi$  is consistent.
- T  F If  $\varphi$  is a tautological conditional, then the antecedent of  $\varphi$  is inconsistent.
- T  F If X is an inconsistent set of SL WFFs, then no member of X is a tautology.
- T  F " $(A \vee \sim(B \& C))$ " is not logically equivalent to " $((C \& B) \rightarrow A)$ ".
- T  F If the premises of an argument are all false, then that argument is not valid.
- T  F No valid argument has a false conclusion.
- T F The following argument can be shown to be valid in SL:  
"If someone is clever, then Rex is clever. Someone is clever. So, Rex is clever."
- T  F "Aristotle said that" is a truth functional connective.
- T F A lexically ambiguous word has more than one meaning in a language.
- T  F Every contingent SL WFF contains at least one connective.

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2. (20 marks)

For each of the following:

Circle "tautology" if it is a WFF of SL that is a tautology.

Circle "contingent" if it is a contingent WFF of SL.

Circle "inconsistent" if it is an inconsistent WFF of SL.

Otherwise, don't circle anything.

tautology	$((D \rightarrow A) \vee (B \& \sim C))$ contingent	inconsistent
tautology	$A \vee A$ contingent	inconsistent
tautology	$((A \& (B \vee C)) \leftrightarrow ((A \vee B) \& (A \vee C)))$ contingent	inconsistent
tautology	$((A \& B) \rightarrow ((A \vee C) \& (B \vee A)))$ contingent	inconsistent
tautology	$((A \rightarrow \sim A) \vee (\sim B \& A))$ contingent	inconsistent
tautology	$((A \& \sim A) \rightarrow (B \& \sim C))$ contingent	inconsistent
tautology	$((A \rightarrow B) \vee (B \rightarrow A))$ contingent	inconsistent
tautology	$((((C \rightarrow B) \rightarrow C) \rightarrow C) \rightarrow C)$ contingent	inconsistent
tautology	$((((C \rightarrow B) \rightarrow C) \rightarrow C)$ contingent	inconsistent
tautology	$(\sim(A \vee B) \leftrightarrow (\sim A \& \sim B))$ contingent	inconsistent

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3. (15 marks)

*Which of the following is a valid argument?*

*Circle "Yes" if it is a valid argument.*

*Circle "No" if it is not a valid argument.*

Yes  No  
If John is happy, then Mary is happy.  
If Martha is happy, then Sue is happy.  
If Sue is happy, then Archibald is happy.  
So, if John is happy, then Archibald is happy.

Yes No  
Hong Kong is a city.  
Hong Kong is not a city.  
Therefore, 4 is a prime number.

Yes No  
If that is a gift, then I am happy.  
So, if that is a gift, then I am happy.

Yes No  
If I drank Coca-Cola then I will feel sick.  
If I drank water then I will feel sick.  
Either I drank Coca-Cola or I drank water.  
So, I will feel sick.

Yes  No  
If you feel sick, then go to the doctor.  
You feel sick.  
So, go to the doctor.

4. (10 marks)

Suppose that a new one-place connective '@', and a new two-place connective '#' are added to SL. You are informed that:

' $((A\#B) \rightarrow @A) \leftrightarrow \sim(A \vee B)$ ' is inconsistent.

' $(A \rightarrow @B)$ ' entails ' $\sim A$ '.

Fill in the truth tables for '#' and '@':

A	@A
T	F
F	F

A	B	(A#B)
T	T	F
T	F	F
F	T	F
F	F	T

Circle 'T' if the statement is true. Circle 'F' if the statement is false:

- T  F '( $@A \vee @B$ )' is contingent.  
 T F ' $@A, (A \rightarrow B) \models B$ ' is a valid sequent.  
 T F ' $(B\#A)$ ' is logically equivalent to ' $(A\#B)$ '.

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5. (10 marks)

Translate the following statements and arguments into SL.

Preserve as much structure as possible.

Use the following translation scheme:

A: Aristotle is late.

P: Plato is at home.

S: Socrates is at home.

L: Aristotle is shopping.

(a) If Aristotle is late, then Plato is at home only if Socrates is too.

$$(A \rightarrow (P \rightarrow S))$$

(b) Whenever Aristotle is shopping, Plato is not at home.

$$(L \rightarrow \neg P)$$

(c) Although Socrates and Plato are at home, Aristotle is shopping.

$$((S \& P) \& L)$$

(d) If Socrates is not at home, then Aristotle is shopping. But if Aristotle is shopping, Plato is at home if Aristotle is late. Therefore, Socrates is at home if and only if Plato is not at home.

$$(\neg S \rightarrow L), (L \rightarrow (A \rightarrow P)) \vdash (S \leftrightarrow \neg P)$$

(e) Whether Aristotle is late or not, Socrates is at home. It is not the case the Socrates and Plato are both at home. Aristotle is not late but he is shopping. So, Plato is not at home.

$$((A \vee \neg A) \rightarrow S), \neg (S \& P), (\neg A \& L) \vdash \neg P$$

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6. (10 marks)

Translate the following statements and arguments into MPL.

Preserve as much structure as possible.

Use the following translation scheme:

a: Archibald

b: Boris

Nx: x is nearby.

Gx: x is good.

(a) If anything is good, Archibald is.

$$(\exists x Gx \rightarrow Ga)$$

(b) If something is nearby, then it is good.

$$\forall x (Nx \rightarrow Gx)$$

(c) Archibald is good, but Boris is nearby. There is nothing good that is nearby. So Boris is not good.

$$(Ga \ \& \ Nb), \ \neg \exists x (Gx \ \& \ Nx) \ \vdash \ \neg Gb$$

(d) Something is either good or nearby but not both. Archibald is neither good nor nearby. Boris is not nearby. Therefore, Boris is good.

$$\exists x ((Gx \vee Nx) \ \& \ \neg (Gx \ \& \ Nx)), \\ (Ga \vee Na), \ \neg Nb \ \vdash \ Gb$$

(e) If Boris is good, then everything is. If Boris is not nearby, then nothing is. Therefore, either Boris is not good or something is nearby.

$$(Gb \rightarrow \forall x Gx), \ (\neg Nb \rightarrow \neg \exists x Nx) \ \vdash \ (\neg Gb \vee \exists x Nx)$$

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