

Midterm Test  
Elementary Logic II  
4 March 2008

---

Please write clearly.  
You have 60 minutes to complete this test.

Student ID Number Catherine Connolly & Arthur Chin

Mark \_\_\_\_\_%

1. (30 marks)

True or false?

Circle 'T' if the statement is true.

Circle 'F' if the statement is false.

Assume that  $\varphi$  and  $\psi$  are WFFs of SL.

- T  F Any system that is complete is also sound.
- T F  $(\varphi \& \sim \psi)$  entails  $\varphi$ .
- T  F If  $\varphi$  is a contradiction, then  $\varphi$  is not derivable in our natural deduction system.
- T F Our natural deduction system can be used to show that a sequent is valid.
- T  F If rule PC is removed, then the resulting system would not be complete.
- T  F If rule PC is removed, then the resulting system would not be sound.
- T  F Any system that is sound is also complete.
- T  F Every correct derivation uses the rule of assumption.
- T F There is a correct derivation with exactly 11 lines.
- T  F If  $\varphi$  is a tautology, then  $\psi$  is derivable in our natural deduction system.

/30

(2) (40 marks) Show each of the following:

(a)  $\vdash (A \leftrightarrow A)$

- 1    ①  $A$                      $A$
- ②  $(A \rightarrow A)$         1  $\rightarrow I$
- ③  $(A \leftrightarrow A)$         2  $\leftrightarrow I$

(b)  $\vdash (A \vee \sim A)$

- 1 ①  $\neg(A \vee \neg A)$        $A$
- 2 ②  $A$        $A$
- 2 ③  $(A \vee \neg A)$       2 VI
- 1, 2 ④  $((A \vee \neg A) \& \neg(A \vee \neg A))$       1, 3  $\&I$
- 1 ⑤  $\neg A$       2, 4  $\neg I$
- 1 ⑥  $(A \vee \neg A)$       5 VI
- 1 ⑦  $((A \vee \neg A) \& \neg(A \vee \neg A))$       1, 6  $\&I$
- ⑧  $(A \vee \neg A)$       1, 7  $\neg E$

(c)  $\vdash ((A \& B) \vee (A \& \sim B))$

- 1 ①  $A$        $A$
- 2 ②  $\neg((A \& B) \vee (A \& \sim B))$        $A$
- 3 ③  $B$        $A$
- 1, 3 ④  $(A \& B)$       1, 3  $\&I$
- 1, 3 ⑤  $((A \& B) \vee (A \& \sim B))$       4, VI
- 1, 2, 3 ⑥  $((A \& B) \vee (A \& \sim B) \& \neg((A \& B) \vee (A \& \sim B)))$       2, 5  $\&I$
- 1, 2 ⑦  $\neg B$       2, 6  $\negI$
- 1, 2 ⑧  $(A \& \sim B)$       1, 7  $\&I$
- 1, 2 ⑨  $((A \& B) \vee (A \& \sim B))$       8 VI
- 1, 2 ⑩  $((A \& B) \vee (A \& \sim B) \& \neg((A \& B) \vee (A \& \sim B)))$       2, 9  $\&I$
- 1 ⑪  $((A \& B) \vee (A \& \sim B))$       2, 10  $\negE$

(d)  $(A \leftrightarrow B) \vdash (A \rightarrow (A \vee B))$

1	①	$(A \leftrightarrow B)$	A
1	②	$(A \rightarrow B) \ \& \ (B \rightarrow A)$	1 $\leftrightarrow$ E
1	③	$(A \rightarrow B)$	2 $\&$ E
4	④	A	A
1, 4	⑤	B	3, 4 $\rightarrow$ E
1, 4	⑥	$(A \vee B)$	5 $\vee$ I
1	⑦	$(A \rightarrow (A \vee B))$	4, 6 $\rightarrow$ I

(e)  $(A \rightarrow B) \vdash (\sim B \rightarrow \sim A)$

1	①	$(A \rightarrow B)$	A
2	②	A	A
1, 2	③	B	1, 2 $\rightarrow$ I
4	④	$\sim B$	A
1, 2, 4	⑤	$(B \ \& \ \sim B)$	3, 4 $\&$ I
1, 4	⑥	$\sim A$	2, 5 $\sim$ I
1	⑦	$(\sim B \rightarrow \sim A)$	4, 6 $\rightarrow$ I

(3) (10 marks) Circle your answer.

Suppose our natural deduction system is revised by adding the following rule:  
(NR) if you have derived  $(\varphi \& \psi)$ , then you can write down  $(\varphi \vee \psi)$ , depending on everything  $(\varphi \& \psi)$  depends on.

Is the revised system sound?     YES    NO

Is the revised system complete?     YES    NO

/10

(4) (10 marks) Circle your answer.

Suppose rule  $\&E$  is removed from our natural deduction system.

Is the revised system sound?     YES    NO

Is the revised system complete?    YES     NO

/10

(5) (10 marks) Circle your answer.

Suppose our natural deduction system is revised by adding the following rule:  
(NR1) if you have derived  $(\varphi \rightarrow \psi)$ , then you can write down  $(\sim\varphi \vee \psi)$ , depending on everything  $(\varphi \rightarrow \psi)$  depends on.

Is the revised system sound?     YES    NO

Is the revised system complete?     YES    NO

/10