

Problem Set 1
Elementary Logic II
Due: 20 February 2008

Submit your problem set to Ms. Loletta Li in Main Building 312. Make sure your problem set is timestamped. Do not submit assignments by email. Late penalty: 10% for each day late. This problem set will not be accepted after 28 February.

Answer the questions on the problem set itself. Write neatly. If the grader cannot read your handwriting, you will not receive credit.

Be sure that all pages of the assignment are securely stapled together.

Check the course bulletin board for announcements about the assignment.

Do your own work.

If you copy your problem set, or permit others to copy, you may fail the course.

Name Catherine Carroll and Arthur Chin

Student ID Number _____

email _____

Score: _____ of 84 marks.

Due 20 February 2008 by 4:00PM.

1. (20 marks)

True or false?

Circle 'T' if the statement is true.

Circle 'F' if the statement is false.

For this question, you should assume that φ is a WFF of SL.

T F φ entails φ .

T F There is no longest derivation in our natural deduction system.

T F Every correct derivation uses the rule of assumption.

T F In a derivation, it is possible to assume φ without deriving φ .

T F " $(A \rightarrow \sim A)$ " is an explicit contradiction.

T F If φ is a tautology, then φ is derivable in our natural deduction system.

T F Our natural deduction system can be used to show that a sequent is valid.

T F Any system that is complete is also sound.

T F If rules $\leftrightarrow I$ and $\leftrightarrow E$ are removed, then the resulting system would not be complete.

T F One way to show that a sequent is not valid is to make a truth table.

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(2) (40 marks) Show each of the following:

(a) $\vdash (A \rightarrow (B \rightarrow A))$

(b) $A, \sim B \vdash \sim(A \rightarrow B)$

(c) $A \vdash ((A \vee B) \& (A \vee \sim B))$

(d) $(A \rightarrow B), (C \rightarrow D) \vdash ((A \& C) \rightarrow (B \& D))$

$$(e) \sim A, (A \leftrightarrow B) \vdash \sim B$$

$$(f) (A \rightarrow B) \vdash (\sim A \vee B)$$

$$(g) (A \vee (B \& C)) \vdash ((A \vee B) \& (A \vee C))$$

$$(h) \sim(A \& B) \vdash (\sim A \vee \sim B)$$

(i) $\sim\sim A \vdash A$

(j) $(\sim A \rightarrow B) \vdash (A \vee B)$

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(3) (6 marks) Circle your answer.

Suppose our natural deduction system is revised by adding the following rule:
(NR) if you have derived $(\varphi \vee \psi)$, then you can write down $(\varphi \& \psi)$, depending on everything $(\varphi \vee \psi)$ depends on.

Is the revised system sound? YES NO
Is the revised system complete? YES NO

/6

(4) (6 marks) Circle your answer.

Suppose rule PC is removed from our natural deduction system.

Is the revised system sound? YES NO
Is the revised system complete? YES NO

/6

(5) (6 marks) Circle your answer.

Suppose rule $\sim E$ is removed from our natural deduction system.

Is the revised system sound? YES NO
Is the revised system complete? YES NO

/6

(6) (6 marks) Circle your answer.

Suppose our natural deduction system is revised by adding the following rule:

(NR2) if you have derived $(\varphi \rightarrow \psi)$, then you can write down $(\varphi \leftrightarrow \psi)$, depending on everything $(\varphi \rightarrow \psi)$ depends on.

Is the revised system sound? YES NO
Is the revised system complete? YES NO

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