Problem Set 2

Elementary Logic II

Due: 16 April 2008

Submit your problem set to Ms. Loletta Li in Main Building 312. Make sure your problem set is timestamped. Do not submit assignments by email. Late penalty: 10% for each day late. This problem set will not be accepted after 18 April 2008.

Answer the questions on the problem set itself. Write neatly. If the grader cannot read your handwriting, you will not receive credit.

Be sure that all pages of the assignment are securely stapled together.

Check the course bulletin board for announcements about the assignment.

Do your own work.
If you copy your problem set, or permit others to copy, you may fail the course.

Name ______________________________

Student ID Number ______________________________

email ______________________________

Score: ______________

Due **16 April 2008** by **4:00PM**.

1. (20 marks)

True or false?
Circle 'T' if the statement is true.
Circle 'F' if the statement is false.
For this question, you should assume that φ is a WFF of MPL, and the derivations are in our natural deduction system for MPL.

T F φ is true under at least one interpretation.

T F If c is a constant and v is a variable, then $\varphi v/c$ is a well-formed formula of MPL.

T F Some correct derivations have more than 1 million lines.

T F "$\exists x Ax$" is valid MPL formula.

T F φ might be a well-formed formula of SL.

T F If rule $\leftrightarrow I$ is removed, then the resulting system would not be sound.

T F Some MPL sequents are not valid.

T F If φ is valid, then there is a derivation of φ with no dependencies.

T F "$\forall x(\exists x Rx \& Cx)$" is a well-formed formula of MPL.

T F The truth table method can always determine whether or not an MPL formula is valid.

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(2) (40 marks) Show each of the following using natural deduction:

(a) $\forall x(Ax \rightarrow Bx), \sim Ba \vdash \sim Aa$

(b) $\forall x(Ax \leftrightarrow Bx), \forall x \sim (Bx \vee \sim Cx) \vdash \sim Ab$

(c) $\sim \exists x \sim Ax \vdash \forall x Ax$

(d) $\forall x(Ax \& Bx) \vdash (\forall x Ax \& \forall x Bx)$

(e) $\forall x(Ax \rightarrow \exists yBy) \vdash (\exists xAx \rightarrow \exists yBy)$

(f) $\forall x(Ax \vee Gx) \vdash (\forall xAx \vee \exists xGx)$

(g) $\vdash \exists x(\exists yAy \rightarrow Ax)$

(h) $\forall x(Ax \vee (Bx \vee Cx)), \forall x(Bx \rightarrow Dx), \forall x(Cx \rightarrow Dx) \vdash \forall x(Ax \vee Dx)$

(i) $Ac, Bc, \forall x(Bx \to Cx) \vdash \exists x(Ax \& Cx)$

(j) $\forall x(Ax \to Bx) \vdash \sim\exists x(Ax \& \sim Bx)$

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(3) (6 marks) Circle your answer.

Suppose the MPL natural deduction system is revised by adding the following rule: (NR) for any variable v and constant c, if you have derived $\forall v(\varphi \vee \psi)$, then you can write down $\varphi v/c$, depending on everything $\forall v(\varphi \vee \psi)$ depends on.

Is the revised system sound? **YES** **NO**

Is the revised system complete? **YES** **NO**

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(4) (6 marks) Circle your answer.

Suppose rule PC is removed from the MPL natural deduction system.

Is the revised system sound? **YES** **NO**

Is the revised system complete? **YES** **NO**

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(5) (6 marks) Circle your answer.

Suppose the MPL natural deduction system is revised by adding the following rule: (NR2) If you have derived $\forall v\varphi$, then you can write down $\exists v\varphi$, depending on everything $\forall v\varphi$ depends on.

Is the revised system sound?	**YES**	**NO**
Is the revised system complete?	**YES**	**NO**

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(6) (6 marks) Circle your answer.

Suppose rule $\forall E$ is removed from the MPL natural deduction system.

Is the revised system sound?	**YES**	**NO**
Is the revised system complete?	**YES**	**NO**

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(7) (16 marks)

Translate the following two arguments into MPL. (The arguments come from Lewis Carroll's Symbolic Logic.*) Be sure to write down your translation schemes. Then, for each argument, either show that the argument is valid using natural deduction, or show that the argument is not valid by giving an appropriate interpretation.*

1. Babies are illogical. Nobody is despised who can manage a crocodile. Illogical people are despised. Babies are people. Therefore, babies cannot manage crocodiles.

2. No terriers wander among the signs of the zodiac. Something that does not wander among the signs of the zodiac, is not a comet. Nothing but a terrier has a curly tail. Therefore, some comets have curly tails.

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