Natural Deduction for Monadic Predicate Logic
Logic 2510, 12 November 2007

Quantifier Rules

$\forall$E (Universal Quantifier Elimination)
For any variable v and constant c,
if you have derived $\forall v\phi$, then you can write down $\phi v/c$,
depending on everything $\forall v\phi$ depends on.

$\exists$I (Existential Quantifier Introduction)
For any variable v and constant c,
if you have derived $\phi v/c$, then you can write down $\exists v\phi$,
depending on everything $\phi v/c$ depends on.

$\forall$I (Universal Quantifier Introduction)
For any variable v and constant c,
if you have derived $\phi v/c$, and c does not occur in ϕ,
and c does not occur in anything ϕ depends on, then you can
write down $\forall v\phi$, depending on everything $\phi v/c$ depends on.

$\exists$E (Existential Quantifier Elimination)
For any variable v and constant c,
if you have derived $\exists v\phi$, assumed $\phi v/c$, and derived ψ,
and c does not occur in ψ, ϕ, or anything ψ depends on (except $\phi v/c$),
then you can write down ψ a second time, depending on everything
$\exists v\phi$ and the first ψ depend on, except the assumption $\phi v/c$.

(In the above rules, $\phi v/c$ is the result of replacing
each free occurrence of v in ϕ by c. If $\forall v\phi$ or $\exists v\phi$ is a sentence,
then any occurrence of v in ϕ is bound. If an occurrence of v
in a sentence ϕ is bound, then that occurrence of v is bound in
a sentence constructed from ϕ using the connectives and/or quantifiers.
If an occurrence of v is not bound, then that occurrence of v is free.)

Connective Rules and Rule of Assumption

A (Rule of Assumption)
You can write down any MPL sentence, depending on itself.

$\wedge$I (Conjunction Introduction)
If you have derived ϕ and ψ, you can write down $(\phi\wedge\psi)$,
depending on everything ϕ and ψ depend on.

$\wedge$E (Conjunction Elimination)
If you have derived $(\phi\wedge\psi)$, you can write down ϕ or ψ,
depending on everything $(\phi\wedge\psi)$ depends on.

$\rightarrow$I (Conditional Introduction)
If you have assumed ϕ, and you have derived ψ, you can write down $(\phi\rightarrow\psi)$,
depending on everything ψ depends on except ϕ.

$\rightarrow$E (Conditional Elimination or Modus Ponens)
If you have derived $(\phi\rightarrow\psi)$ and ϕ, you can write down ψ,
depending on everything $(\phi\rightarrow\psi)$ and ϕ depend on.

$\neg$I (Negation Introduction)
If you have assumed ψ, and you have derived $(\phi\wedge\neg\phi)$, then you can write down $\neg\psi$,
depending on everything $(\phi\wedge\neg\phi)$ depends on except ψ.

¬E (Negation Elimination)
If you have assumed $\neg\psi$, and you have derived $(\phi\wedge\neg\phi)$, then you can write down ψ,
depending on everything $(\phi\wedge\neg\phi)$ depends on except $\neg\psi$.

∨I (Disjunction Introduction)
If you have derived ϕ, you can write down $(\phi\vee\psi)$ or $(\psi\vee\phi)$,
depending on everything ϕ depends on.
(ψ is any MPL sentence.)

∨E (Disjunction Elimination or Disjunctive Syllogism)
If you have derived $(\phi\vee\psi)$ and $\neg\psi$, you can write down ϕ,
depending on everything $(\phi\vee\psi)$ and $\neg\psi$ depend on.

If you have derived $(\phi\vee\psi)$ and $\neg\phi$, you can write down ψ,
depending on everything $(\phi\vee\psi)$ and $\neg\phi$ depend on.

PC (Proof by Cases)
If you have derived $(\phi\vee\psi)$ and $(\phi\rightarrow\alpha)$ and $(\psi\rightarrow\beta)$, then you can write down $(\alpha\vee\beta)$,
depending on everything $(\phi\vee\psi)$ and $(\phi\rightarrow\alpha)$ and $(\psi\rightarrow\beta)$ depend on.

↔I (Biconditional Introduction)
If you have derived $((\Phi\rightarrow\Psi)\wedge(\Psi\rightarrow\Phi))$, you can write down $(\Phi\leftrightarrow\Psi)$,
depending on everything $((\Phi\rightarrow\Psi)\wedge(\Psi\rightarrow\Phi))$ depends on.

↔E (Biconditional Elimination)
If you have derived $(\Phi\leftrightarrow\Psi)$ then you can write down $((\Phi\rightarrow\Psi)\wedge(\Psi\rightarrow\Phi))$
depending on everything $(\Phi\leftrightarrow\Psi)$ depends on.