Natural Deduction for the Sentential Calculus
Logic 2510, 12 September 2007 (Revised)

A (Rule of Assumption)
You can write down any SC sentence, depending on itself.

$\wedge$I (Conjunction Introduction)
If you have derived ϕ and ψ, you can write down $(\phi \wedge \psi)$, depending on everything ϕ and ψ depend on.

$\wedge$E (Conjunction Elimination)
If you have derived $(\phi \wedge \psi)$, you can write down ϕ or ψ, depending on everything $(\phi \wedge \psi)$ depends on.

$\rightarrow$I (Conditional Introduction)
If you have assumed ϕ, and you have derived ψ, you can write down $(\phi \rightarrow \psi)$, depending on everything ψ depends on except ϕ.

$\rightarrow$E (Conditional Elimination or Modus Ponens)
If you have derived $(\phi \rightarrow \psi)$ and ϕ, you can write down ψ, depending on everything $(\phi \rightarrow \psi)$ and ϕ depend on.

$\neg$I (Negation Introduction)
If you have assumed ψ, and you have derived $(\phi \wedge \neg \phi)$, then you can write down $\neg \psi$, depending on everything $(\phi \wedge \neg \phi)$ depends on except ψ.

$\neg$E (Negation Elimination)
If you have assumed $\neg \psi$, and you have derived $(\phi \wedge \neg \phi)$, then you can write down ψ, depending on everything $(\phi \wedge \neg \phi)$ depends on except $\neg \psi$.

$\vee$I (Disjunction Introduction)
If you have derived ϕ, you can write down $(\phi \vee \psi)$ or $(\psi \vee \phi)$, depending on everything ϕ depends on.
(ψ is any SC sentence.)

$\vee$E (Disjunction Elimination or Disjunctive Syllogism)
If you have derived $(\phi \vee \psi)$ and $\neg \psi$, you can write down ϕ, depending on everything $(\phi \vee \psi)$ and $\neg \psi$ depend on.

If you have derived $(\phi \vee \psi)$ and $\neg \phi$, you can write down ψ, depending on everything $(\phi \vee \psi)$ and $\neg \phi$ depend on.

PC (Proof by Cases)
If you have derived $(\phi \vee \psi)$ and $(\phi \rightarrow \alpha)$ and $(\psi \rightarrow \beta)$, then you can write down $(\alpha \vee \beta)$, depending on everything $(\phi \vee \psi)$ and $(\phi \rightarrow \alpha)$ and $(\psi \rightarrow \beta)$ depend on.

$\leftrightarrow$I (Biconditional Introduction)
If you have derived $((\phi \rightarrow \psi) \wedge (\psi \rightarrow \phi))$, you can write down $(\phi \leftrightarrow \psi)$, depending on everything $((\phi \rightarrow \psi) \wedge (\psi \rightarrow \phi))$ depends on.

$\leftrightarrow$E (Biconditional Elimination)
If you have derived $(\phi \leftrightarrow \psi)$ then you can write down $((\phi \rightarrow \psi) \wedge (\psi \rightarrow \phi))$ depending on everything $(\phi \leftrightarrow \psi)$ depends on.