Predicate Logic

Syntax of Predicate Logic (PL)

PL contains only the following symbols:

1. *Predicate letters* :
   Zero-place predicate letters: A^0, B^0, C^0, etc.
   One-place predicate letters: A^1, B^1, C^1, etc.
   Two-place predicate letters: A^2, B^2, C^2, etc.
   Three-place predicate letters: A^3, B^3, C^3, etc.
   ...
   (That is, for any n, n-place predicate letters: A^n, B^n, C^n, etc.)
   If a numerical subscript is added to a predicate letter, the result is a predicate letter, for example, B^2_3, C^0_{323}, A^1_1.

2. *Constants* : $a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p, q, r, s, t$
   If a numerical subscript is added to a constant, the result is a constant, for example, a_1, b_{23}, etc.

3. *Variables* : u, v, w, x, y, z
   If a numerical subscript is added to a variable, the result is a variable, for example, u_1, x_2, etc.

4. Five *sentential connectives* : $\neg$, $\wedge$, $\vee$, $\rightarrow$, $\leftrightarrow$

5. *Right and left parentheses* : ( )

The rules of formation of PL are as follows:

1. Any n-place predicate letter followed by a n constants is a sentence.

2. If φ is a sentence then $\neg\varphi$ is a sentence.

3. If φ and ψ are sentences, then $(\varphi \wedge \psi)$, $(\varphi \vee \psi)$, $(\varphi \rightarrow \psi)$, $(\varphi \leftrightarrow \psi)$ are sentences.

4. If φ is a sentence that contains any constant ω, and does not contain variable β, then the expression that results by replacing one or more occurrences of ω with β and then attaching "$\exists\beta$" or "$\forall\beta$" to the front, is also a sentence.

5. Nothing else is a sentence.

Abbreviation

For convenience, the superscripts need not be written.
For example, we can write "Aa" to abbreviate "A^1a", "Amn" for "A^2mn" and so on.
Note that the "A" in "Aa" abbreviates a different predicate letter than the "A" in "Amn".

Examples

For example, the following are sentences of PL:

Ab
Pmn
$\forall x Axy$
$(\exists x Ex \wedge \exists x Rx)$
B
$\forall x \forall y \exists z (Bxyz \vee \forall v Cvx)$
$\forall x \exists y (Sx \rightarrow Lxy)$
$\exists y \forall x (Sx \rightarrow Lxy)$

However, these are not sentences of PL:

$\forall x Ab$
$\forall y \exists y Axy$
$\forall x (\exists y Cy \rightarrow \exists x Bxy)$

Can you see why?