

Homework 2

Due Thursday 20 September in class

(1) Show each of the following:

(a) $(\neg P \rightarrow Q) \vdash (P \vee Q)$

(b) $\neg\neg P \vdash P$

(c) $\vdash (P \rightarrow \neg\neg P)$

(d) (question deleted)

(e) $((P \wedge Q) \vee (P \wedge R)) \vdash (P \wedge (Q \vee R))$

(f) $\neg(P \wedge Q) \vdash (\neg P \vee \neg Q)$

(g) $\neg(P \vee Q) \vdash (\neg P \wedge \neg Q)$

(h) $(P \rightarrow Q), (R \rightarrow Q) \vdash ((P \vee R) \rightarrow Q)$

(i) $(P \rightarrow (Q \rightarrow R)) \vdash ((P \rightarrow Q) \rightarrow (P \rightarrow R))$

(j) $\vdash ((\neg Q \rightarrow Q) \rightarrow Q)$

(2) Suppose the Rule PC (Proof by Cases) is removed from our natural deduction system. Is there anything that can no longer be proved, that could have been proved before? If so, give an example. If not, explain why not.

(3) Suppose the following rule is added to our natural deduction system:
(MT) if you have derived $\neg\psi$ and $(\varphi \rightarrow \psi)$, then you can write down $\neg\varphi$, depending on everything $\neg\psi$ and $(\varphi \rightarrow \psi)$ depend on.

Is there anything that can be proved now, that could not be proved before? If so, give an example. If not, explain why not.