

Homework 4

Due Wednesday 7 November

- (1) Determine whether or not the following argument is valid by first translating the argument into the sentential calculus, and then determining whether or not the resulting SC argument is valid. Be sure to write down your translation scheme.

If Liz is not angry but Mark is, then Rex is angry. If Rex is angry, neither Mark nor Liz is. Mark is angry if and only if Liz is not angry. Therefore, Liz is angry, but Rex and Mark are not.

- (2) Suppose that Δ is a set of SC sentences such that every finite subset of Δ is consistent, and for every sentence φ , either φ or $\neg\varphi$ is in Δ . Without assuming the Compactness theorem, show that the following are true, for any SC sentences φ and ψ :

- (i) $(\varphi \vee \psi)$ is in Δ if and only if φ is in Δ or ψ is in Δ (or both).
- (ii) $(\varphi \rightarrow \psi)$ is in Δ if and only if φ is not in Δ or ψ is in Δ (or both).
- (iii) $(\varphi \leftrightarrow \psi)$ is in Δ if and only if both φ and ψ are in Δ or neither of them is in Δ .

- (3) In class, we noted that the following corollary can be proved using the Compactness theorem. Indeed McGee proves the corollary using the Compactness theorem on page 4 of chapter 9.

Corollary: If φ is a logical consequence of a set of sentences Δ , then φ is a logical consequence of some finite subset of Δ .

Compactness theorem: A set Γ of SC sentences is consistent if and only if every finite subset of Γ is consistent.

Can the Compactness theorem be proved using the Corollary? If yes, then show how. If not, then explain why not.

- (4) Suppose that Δ is a set of sentences such that every sentence derivable from Δ is in Δ , and for each sentence φ , either φ or $\neg\varphi$ is in Δ . Suppose also that Δ is not the set of all sentences. Show that Δ is a complete story.