

Homework 5

Due Monday 19 November

- (1) Determine whether or not the following argument is valid by first translating the argument into Monadic Predicate Logic, and then determining whether or not the resulting MPL argument is valid.

No one who likes cheese is afraid of heights.

No one who is very well-dressed is afraid of heights.

Liars are very well-dressed.

Therefore, liars don't like cheese.

- (2) Show the following using natural deduction:

(a) $\forall x(Mx \rightarrow Gx), \exists x(Fx \wedge Mx) \vdash \exists x(Fx \wedge Gx)$

(b) $\forall x(Bx \rightarrow Ax) \vdash (\exists xBx \rightarrow \exists xAx)$

(c) $\exists x\neg Gx \vdash \neg\forall xGx$

(d) $\forall x((Cx \vee Bx) \rightarrow Ax), \forall x\neg Ax \vdash \forall x\neg Cx$

(e) $\exists x(Ax \vee Bx) \vdash (\exists xAx \vee \exists xBx)$

- (3) Determine whether the following sequents are valid. If a sequent is valid, write "Valid". If not, give an interpretation which shows that the sequent is not valid.

(a) $(\forall xPx \vee \forall xQx) \models \forall x(Px \vee Qx)$

(b) $\forall x(Px \vee Qx) \models (\forall xPx \vee \forall xQx)$

(c) $\forall x(Px \rightarrow Qx), \exists x(Qx \rightarrow Rx), Pa \models Ra$

(d) $\forall xPx, \exists xQx \models \exists x(Px \wedge Qx)$

(e) $Pa, \neg\exists x\neg(Px \rightarrow Qx) \models \exists xQx$

(f) $\forall x(Px \vee Qx) \models \neg\exists x(Px \wedge \neg Qx)$

(g) $\exists x(Px \wedge Qx), \neg Pa \models \neg Qa$

(h) $(\forall x(Px \rightarrow Qx) \rightarrow \exists y\neg Ry), \exists x\neg Px \models (\forall xRx \rightarrow \exists y(Py \wedge Qy))$