

Midterm Test
Elementary Logic
2 March 2010
version 1.2A

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Please write clearly.
You have 60 minutes to complete this test.

Mark _____ %

1.5 marks each

1. (15 marks) True or False? (φ and ψ are SL WFFs.)

- F The main connective of " $(\sim \sim A \rightarrow B)$ " is " \rightarrow ".
- F Some valid arguments are not sound arguments.
- T F If the premises and conclusion of an argument are all true, then the argument is valid.
- F " $\sim A \vee B$ " is an expression of SL.
- F Whenever " $(A \vee B)$ " is true, " $(\sim A \rightarrow B)$ " is also true.
- T F Every SL WFF contains at least one connective.
- T F Every SL tautology contains at least two connectives.
- T F If $(\varphi \rightarrow \psi)$ is inconsistent then φ is inconsistent.
- F If an argument is sound then its conclusion is true.
- F The scope of "&" in " $(\sim(\sim P \& Q) \rightarrow P)$ " is " $(\sim P \& Q)$ ".

/15

2. (5 marks) Write down a valid SL sequent without using any connectives :

$A \vDash A$

/5

3. (16 marks) Fill in the blanks with an SL WFF to make correct truth tables. Each WFF you write down must be a biconditional.

a.

C	B	A	$((C \vee \neg C) \leftrightarrow (A \& B))$
T	T	T	T
T	T	F	F
T	F	T	F
T	F	F	F
F	T	T	T
F	T	F	F
F	F	T	F
F	F	F	F

b.

A	B	C	$(A \leftrightarrow \neg A)$
T	T	T	F
T	T	F	F
T	F	T	F
T	F	F	F
F	T	T	F
F	T	F	F
F	F	T	F
F	F	F	F

c.

A	B	$(A \leftrightarrow B)$
T	T	T
T	F	F
F	T	F
F	F	T

d.

A	$((A \& \neg A) \leftrightarrow A)$
T	F
F	T

4 marks each

/16

1 mark be deducted for any 2 missing/extra bracket (2 marks max.)

3 marks each

1 mark be deducted for any single mistake except 0.5 mark be deducted for missing / extra bracket (0.5 marks max.)

4. (15 marks) Make a truth table for each of the following WFFs of SL.

a. $(\sim A \rightarrow (B \leftrightarrow B))$

A	B	$(\sim A \rightarrow (B \leftrightarrow B))$
T	T	T
T	F	T
F	T	T
F	F	T

b. $((A \vee C) \& (A \vee B))$

A	B	C	$((A \vee C) \& (A \vee B))$
T	T	T	T
T	T	F	T
T	F	T	T
T	F	F	T
F	T	T	T
F	T	F	T
F	F	T	F
F	F	F	F

c. $((A \vee \sim \sim B) \rightarrow \sim A)$

A	B	$((A \vee \sim \sim B) \rightarrow \sim A)$
T	T	F
T	F	F
F	T	T
F	F	T

d. $\sim((A \rightarrow B) \rightarrow (A \rightarrow C))$

A	B	C	$\sim((A \rightarrow B) \rightarrow (A \rightarrow C))$
T	T	T	F
T	T	F	T
T	F	T	F
T	F	F	F
F	T	T	F
F	T	F	F
F	F	T	F
F	F	F	F

e. $((A \& \sim B) \& \sim \sim C) \& \sim \sim \sim D$

A	B	C	D	$((A \& \sim B) \& \sim \sim C) \& \sim \sim \sim D$
T	T	T	T	F
T	T	T	F	F
T	T	F	T	F
T	T	F	F	F
T	F	T	T	F
T	F	T	F	F
T	F	F	T	F
T	F	F	F	F
F	T	T	T	F
F	T	T	F	F
F	T	F	T	F
F	T	F	F	F
F	F	T	T	F
F	F	T	F	F
F	F	F	T	F
F	F	F	F	F

/15

5. (10 marks) Which of the following is a valid argument? Circle "Yes" if it is a valid argument. Circle "No" if it is not a valid argument.

- Yes No Nelson never chased Napoleon.
So, Napoleon was never chased by Nelson. 2 marks each
- Yes No The population of Canada is increasing.
The population of Canada is not increasing.
Therefore, the population of Vancouver is increasing.
- Yes No If you are late, you will blame me.
You are late.
So, you will blame me.
- Yes No If Descartes was happy, Socrates was happy.
Descartes was not happy.
Thus, Socrates was not happy.
- Yes No I don't know that I am not dreaming.
If I don't know that I am awake, then I don't know that I am not dreaming.
Therefore, I don't know that I am awake.

/10

6. (10 marks) For each of the following: Circle "tautology" if it is a WFF of SL that is a tautology. Circle "contingent" if it is a contingent WFF of SL. Circle "inconsistent" if it is an inconsistent WFF of SL. Otherwise, don't circle anything.

- tautology $((\sim B \leftrightarrow A) \& (\sim A \leftrightarrow B))$ contingent inconsistent
- tautology $((A \& A) \rightarrow (C \vee B)) \rightarrow \sim \sim C$ contingent inconsistent
- tautology $((\sim B \rightarrow (A \rightarrow A)) \& C)$ contingent inconsistent
- tautology $((C \vee \sim B) \rightarrow (C \& \sim B))$ contingent inconsistent
- tautology $(\sim(B \& ((A \& C) \rightarrow (A \vee C))))$ contingent inconsistent

2 marks each

not a WFF

/10

7. (10 marks) Suppose that a new two-place connective '#' is added to SL. You are informed that " $(A \# B)$ " entails " $(A \leftrightarrow B)$ ", and that " $(A \rightarrow B)$ " entails " $(A \# B)$ ". If possible, complete the following truth table. If it is not possible to complete the truth table, explain why.

A	B	$(A \# B)$
T	T	
T	F	
F	T	
F	F	

It's not possible to complete the truth table, for if $(A \# B)$ entails $(A \leftrightarrow B)$, $(A \# B)$ will be false when $(A \leftrightarrow B)$ is false, & if $(A \rightarrow B)$ entails $(A \# B)$, $(A \# B)$ will be true when $(A \rightarrow B)$ is true. Now consider the case when A is false & B is true, $(A \leftrightarrow B)$ will be false & $(A \rightarrow B)$ will be true, which means that $(A \# B)$ has to be both false & true at the same time, i.e. contradiction arises.

2 marks each
No mark be deducted for any
missing / extra bracket (1 mark max.)

8. (10 marks)

Translate the following statements into SL, preserving as much structure as possible.
Be sure to write down your translation scheme.

(a) I don't give discounts unless you are my friend.

D: I give discounts

F: You are my friend

$$(\neg F \rightarrow \neg D)$$

(b) I wonder whether the humidity will increase quickly.

Q: I wonder whether the humidity will increase quickly

Q

(c) Provided that the government agrees, you will get a visa.

A: the government agrees

V: You will get a visa

$$(A \rightarrow V)$$

(d) Commercial shipping is viable only if demand is high.

V: Commercial shipping is viable

H: Demand is high

$$(V \rightarrow H)$$

(e) Both John and Mary met Mr Lee, but neither John nor Mary left early.

J: John met Mr Lee

M: Mary met Mr Lee

$((J \& M) \& (\neg K \& \neg N))$

K: John left early

N: Mary left early

/10

3 marks each.

9. (9 marks) Two of the following three statements are false and one is true:

- (a) Lola and Harry both eat only if the potatoes are not spicy.
(b) If Harry does not eat, then the rice is cooked, but if the rice is cooked then Harry does not eat.
(c) If Lola eats then the rice is not cooked.

1. Translate each of the three statements into SL, preserving as much structure as possible. Be sure to write down your translation scheme.

L = Lola eats

(a) $((L \& H) \rightarrow \neg P)$

H = Harry eats

(b) $((\neg H \rightarrow R) \& (R \rightarrow \neg H))$

P = The potatoes are spicy

(c) $(L \rightarrow \neg R)$

R = The rice is cooked

1 mark for each correct WFF,
0.5 mark be deducted for any missing /
extra bracket

2. Which one of the three statements is true? (If there is not enough information to answer the question, explain why.)

The statement "Lola and Harry both eat only if the potatoes are not spicy" is true.

3. Are the potatoes spicy?

(If there is not enough information to answer the question, explain why.)

No, the potatoes are not spicy.

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END OF PAPER