Problem Set 2

Elementary Logic

Due: 14 April 2010

Name ______________________________

Student ID Number ______________________________

Mark ____________%

Due **14 April 2010** by **4:00PM**.
Submit your problem set to Ms. Loletta Li in Main Building 312. Make sure your problem set is timestamped. Do not submit assignments by email. Late penalty: 10% for each day late. This problem set will not be accepted after 16 April.
Answer the questions on the problem set itself. Write neatly. If the grader cannot read your handwriting, you will not receive credit. Be sure that all pages of the assignment are securely stapled together. Check the course bulletin board for announcements about the assignment. Do your own work. If you copy your problem set, or permit others to copy, you may fail the course.

1. (15 marks) *True or false?*
   Circle 'T' if the statement is true.
   Circle 'F' if the statement is false.
   φ and ψ are SL WFFs.

T	F	If φ is an inconsistent conjunction, then each conjunct of φ is inconsistent.
T	F	If $(\psi \& \sim \psi)$ entails φ then φ is consistent.
T	F	If X is a consistent set of MPL WFFs, then every member of X is consistent.
T	F	There is no interpretation under which "$\exists x(Fx \rightarrow Gx)$" is false and "$\forall x(Fx \& Gx)$" is true.
T	F	The following argument can be shown to be valid in SL: "If everyone likes cilantro, then someone likes arugula. Someone dislikes arugula. So, not everyone likes cilantro."
T	F	"$\exists x(Fx \rightarrow (Gx \vee Fx))$" is a valid MPL WFF.
T	F	"$\exists x(Wx \leftrightarrow (Wx \& \exists y Wy)$" is a valid MPL WFF.
T	F	"It is certain that" is a truth functional connective.
T	F	Any inductive argument can be made valid by adding one extra premise.
T	F	"$\exists x Fx$" is consistent with "$\exists x \sim Fx$".

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2. (16 marks)
For each of the following:
Circle "valid" if it is a valid sequent.
Circle "invalid" if it is an invalid sequent.
Otherwise, don't circle anything.

$\forall x(Px \vee Qx), Pa \models \forall x(Pa \vee Qx)$	valid	invalid
$\forall x(Px \vee Qx), (Pa \& Ra) \models Qa$	valid	invalid
$(\forall x Px \rightarrow \forall x Qx) \models \exists x(Px \rightarrow Qx)$	valid	invalid
$(Q \& (P \vee (\sim P \& Q))) \models (P \rightarrow \sim Q)$	valid	invalid
$(P \rightarrow (Q \rightarrow \sim Q)) \models \sim P$	valid	invalid
$(Q \& (Q \vee R)) \models (P \rightarrow Q)$	valid	invalid
$Pa, \forall x(Px \rightarrow Qx) \models Qa$	valid	invalid
$\sim \exists x(Px \& Qx), \sim Pa \models \sim Qa$	valid	invalid

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3. (21 marks)
Translate the following statements and arguments into MPL.
Preserve as much structure as possible.
Use the following translation scheme:

b: Bach
m: Mozart
Hx: x listens to Bach
Px: x plays the harpsichord
Cx: x composed a fugue

(a) If Mozart does not play the harpsichord then neither does Bach.

(b) If there is someone who both listens to Bach and plays the harpsichord, then there is someone who both listens to Bach and composed a fugue.

(c) Mozart plays the harpsichord only if everyone composed a fugue or no one did.

(d) Mozart, who did not compose a fugue, and Bach, who listens to Bach, both composed a fugue.

(e) Whoever composed a fugue plays the harpsichord, and whoever listens to Bach composed a fugue. So whoever listens to Bach plays the harpsichord.

(f) Someone composed a fugue although Mozart didn't.

(g) Everyone who listens to Bach listens to Bach, but someone who composed a fugue did not compose a fugue.

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4. (10 marks)

Give an MPL WFF that is logically equivalent to each of the following WFFs. Your answer must include an existential quantifier if the original WFF contains a universal quantifier, and vice versa. (MPL WFF φ is logically equivalent to MPL WFF ψ if and only if φ entails ψ, and ψ entails φ.)

(a) $\sim\exists x(Fx \rightarrow Gx)$

(b) $\forall x(Ax \& \sim Bx)$

(c) $\exists x(Fx \vee \sim Fx)$

(d) $\sim\exists x(Fx \& Gx)$

(e) $\sim\sim\forall x(\sim Gx \rightarrow \sim Fx)$

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5. (20 marks)

Determine whether the following sequents are valid. If a sequent is valid, write "valid". If not, give an interpretation which shows that the sequent is not valid.

$\forall x(Ax \vee Bx) \models (\forall x Ax \vee \forall x Bx)$

$\sim Cc, \forall x(Ax \rightarrow Bx), \forall x(Bx \rightarrow Cx) \models \sim Ac$

$(\forall x Ax \& \exists x \sim Bx) \models \exists x(Ax \& Bx)$

$\forall x(Ax \vee Bx) \models \exists x Ax$

$\sim\exists x(Ax\&Bx), \sim Ab \models Bb$

$(\forall x(Ax \rightarrow Bx) \rightarrow \exists yCy), \exists xBx \models (\forall xBx \rightarrow \exists y(Ay\& \sim By))$

$Aa, \exists x(Ax \rightarrow Bx) \models \exists xBx$

$\exists x(Ax \rightarrow Bx) \models (\sim\exists xAx \vee \exists xBx)$

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6. (18 marks)
For each of the following, circle either "Yes" or "No".

Is there an interpretation under which all the following MPL WFFs are true?

$\forall y \sim (Ay \vee Cy)$
$\exists y\exists x(\sim Bx \vee (\sim Cx \vee Ay))$
$\forall x(Bx \rightarrow Ax)$
$\sim\forall x \sim \sim(Bx \rightarrow Cx)$

Yes No

Is there a consistent MPL WFF which is false under every interpretation containing more than 1027 elements in its domain?

Yes No

Is there an interpretation under which " $\forall x(Ax \rightarrow Bx)$" is false and " $\forall x(Ax \leftrightarrow Bx)$" is true?

Yes No

Is there a consistent set of 7 MPL WFFs such that each WFF in the set is inconsistent with "$\exists xBx$"?

Yes No

Is there an inconsistent set of 24 MPL WFFs such that each pair of WFFs in the set is consistent?

Yes No

Is there an SL WFF which contains no sentence letters other than "A" and "B", and which is entailed by every SL conjunction?

Yes No

Is there a consistent MPL WFF which is false under every interpretation?

Yes No

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