Problem Set 2

Elementary Logic

Due: 14 April 2010

Name LAM Ka Ho

Student ID Number _____

Mark _____%

Due **14 April 2010** by **4:00PM**.
Submit your problem set to Ms. Loletta Li in Main Building 312. Make sure your problem set is timestamped. Do not submit assignments by email. Late penalty: 10% for each day late. This problem set will not be accepted after 16 April.
Answer the questions on the problem set itself. Write neatly. If the grader cannot read your handwriting, you will not receive credit. Be sure that all pages of the assignment are securely stapled together. Check the course bulletin board for announcements about the assignment. Do your own work. If you copy your problem set, or permit others to copy, you may fail the course.

1. (15 marks) *True or false?* 1 mark each
   Circle 'T' if the statement is true.
   Circle 'F' if the statement is false.
   φ and ψ are SL WFFs.

   T (F) If φ is an inconsistent conjunction, then each conjunct of φ is inconsistent.
   T (F) If $(\psi \& \sim \psi)$ entails φ then φ is consistent.
   (T) F If X is a consistent set of MPL WFFs, then every member of X is consistent.
   (T) F There is no interpretation under which "$\exists x(Fx \rightarrow Gx)$" is false and "$\forall x(Fx \& Gx)$" is true.
   T (F) The following argument can be shown to be valid in SL: "If everyone likes cilantro, then someone likes arugula. Someone dislikes arugula. So, not everyone likes cilantro."
   (T) F "$\exists x(Fx \rightarrow (Gx \vee Fx))$" is a valid MPL WFF.
   T (F) "$\exists x(Wx \leftrightarrow (Wx \& \exists y Wy)$" is a valid MPL WFF.
   both are correct (T) (F) "It is certain that" is a truth functional connective.
   (T) F Any inductive argument can be made valid by adding one extra premise.
   (T) F "$\exists x Fx$" is consistent with "$\exists x \sim Fx$".

/15

2. (16 marks)
For each of the following:
Circle "valid" if it is a valid sequent.
Circle "invalid" if it is an invalid sequent.
Otherwise, don't circle anything.

2 marks each

$\forall x(Px \vee Qx), Pa \models \forall x(Pa \vee Qx)$	(valid)	invalid
$\forall x(Px \vee Qx), (Pa \& Ra) \models Qa$	valid	(invalid)
$(\forall x Px \rightarrow \forall x Qx) \models \exists x(Px \rightarrow Qx)$	(valid)	invalid
$(Q \& (P \vee (\sim P \& Q))) \models (P \rightarrow \sim Q)$	valid	(invalid)
$(P \rightarrow (Q \rightarrow \sim Q)) \models \sim P$	valid	(invalid)
$(Q \& (Q \vee R)) \models (P \rightarrow Q)$	(valid)	invalid
$Pa, \forall x(Px \rightarrow Qx) \models Qa$	(valid)	invalid
$\sim\exists x(Px \& Qx), \sim Pa \models \sim Qa$	valid	(invalid)

/16

3. (21 marks)
Translate the following statements and arguments into MPL.
Preserve as much structure as possible.
Use the following translation scheme:

b: Bach
m: Mozart
Hx: x listens to Bach
Px: x plays the harpsichord
Cx: x composed a fugue

3 marks each
0.5 mark deducted for missing bracket
(1.5 marks max.)

(a) If Mozart does not play the harpsichord then neither does Bach.

$(\sim Pm \rightarrow \sim Pb)$

(b) If there is someone who both listens to Bach and plays the harpsichord, then there is someone who both listens to Bach and composed a fugue.

$(\exists x(Hx \& Px) \rightarrow \exists x(Hx \& Cx))$

(c) Mozart plays the harpsichord only if everyone composed a fugue or no one did.

$(Pm \rightarrow (\forall x Cx \vee \sim\exists x Cx))$

(d) Mozart, who did not compose a fugue, and Bach, who listens to Bach, both composed a fugue.

$((\sim Cm \& Hb) \& (Cm \& Cb))$

OR

$((\sim Cm \& Cm) \& (Hb \& Cb))$

(e) Whoever composed a fugue plays the harpsichord, and whoever listens to Bach composed a fugue. So whoever listens to Bach plays the harpsichord.

$\forall x(Cx \to Px),\ \forall x(Hx \to Cx) \not\models \forall x(Hx \to Px)$

OR

$(\forall x(Cx \to Px) \,\&\, \forall x(Hx \to Cx)) \not\models \forall x(Hx \to Px)$

(f) Someone composed a fugue although Mozart didn't.

$(\exists x Cx \,\&\, \sim Cm)$

(g) Everyone who listens to Bach listens to Bach, but someone who composed a fugue did not compose a fugue.

$(\forall x(Hx \to Hx) \,\&\, \exists x(Cx \,\&\, \sim Cx))$

/21

4. (10 marks)

Give an MPL WFF that is logically equivalent to each of the following WFFs. Your answer must include an existential quantifier if the original WFF contains a universal quantifier, and vice versa. (MPL WFF φ is logically equivalent to MPL WFF ψ if and only if φ entails ψ, and ψ entails φ.)

2 marks each

(a) $\sim\exists x(Fx \to Gx)$ $\forall x \sim(Fx \to Gx)$ OR $\forall x \sim(\sim Fx \vee Gx)$ OR $\forall x(Fx \,\&\, \sim Gx)$

(b) $\forall x(Ax \,\&\, \sim Bx)$ $\sim\exists x \sim(Ax \,\&\, \sim Bx)$ OR $\sim\exists x(\sim Ax \vee Bx)$

(c) $\exists x(Fx \vee \sim Fx)$ $\sim\forall x \sim(Fx \vee \sim Fx)$ OR $\sim\forall x(\sim Fx \,\&\, Fx)$

(d) $\sim\exists x(Fx \,\&\, Gx)$ $\forall x \sim(Fx \,\&\, Gx)$ OR $\forall x(\sim Fx \vee \sim Gx)$.

0.5 mark deducted for missing bracket (1 mark max.)

(e) $\sim\sim\forall x(\sim Gx \to \sim Fx)$ $\sim\exists x \sim(\sim Gx \to \sim Fx)$ OR $\sim\exists x \sim(Gx \vee \sim Fx)$ OR. $\sim\exists x(\sim Gx \,\&\, Fx)$

/10

5. (20 marks)

Determine whether the following sequents are valid. If a sequent is valid, write "valid". If not, give an interpretation which shows that the sequent is not valid.

1.5 marks each

1.5 marks deducted for incorrect interpretation

$\forall x(Ax \vee Bx) \models (\forall x Ax \vee \forall x Bx)$ Domain: All human beings

Invalid

Ax : x is male

Bx : x is female

$\sim Cc, \forall x(Ax \to Bx), \forall x(Bx \to Cx) \models \sim Ac$

Valid

$(\forall x Ax \,\&\, \exists x \sim Bx) \models \exists x(Ax \,\&\, Bx)$ Domain: All human beings

Invalid

Ax : x is warm-blooded

Bx : x is a sheep

$\forall x(Ax \vee Bx) \models \exists x Ax$

Invalid Domain: All mammals

Ax : x is cold-blooded

Bx : x is warm-blooded

* for question 5, many students misunderstood what an interpretation is, here, you are required to provide an interpretation to show that the argument is INVALID, i.e. the premises are <u>true</u> BUT the conclusion is <u>false</u>.

** An interpretation should have only ONE single domain, no matter how many variables are being used. (x & y should NOT have different domain, for they are only variables!)

*** Some students forget to give an interpretation to the constant "a" & "b"!

$\sim\exists x(Ax\&Bx), \sim Ab \models Bb$ Domain: All mammals. Ax: x is cold-blooded; Bx: x has fins; b: a man called "Lam Ka Ho"

Invalid

$(\forall x(Ax \to Bx) \to \exists yCy), \exists xBx \models (\forall xBx \to \exists y(Ay\& \sim By))$ Domain: All mammals. Ax: x is a sheep; Bx: x is warm-blooded; Cx: x has hair

Invalid

$Aa, \exists x(Ax \to Bx) \models \exists xBx$ Domain: All mammals. Ax: x Lays eggs; Bx: x is cold-blooded; a: a Platypus called "Lam Ka Ho"

Invalid

$\exists x(Ax \to Bx) \models (\sim\exists xAx \vee \exists xBx)$ Domain: All human beings. Ax: x is male; Bx: x Can fly

Invalid

/20

6. (18 marks)
For each of the following, circle either "Yes" or "No".

Is there an interpretation under which all the following MPL WFFs are true?

$\forall y \sim (Ay \vee Cy)$
$\exists y\exists x(\sim Bx \vee (\sim Cx \vee Ay))$
$\forall x(Bx \to Ax)$
$\sim\forall x \sim \sim(Bx \to Cx)$

2.5 marks each
0.5 bonus mark for all correct

Yes (No)

Is there a consistent MPL WFF which is false under every interpretation containing more than 1027 elements in its domain?

Yes (No)

Is there an interpretation under which "$\forall x(Ax \to Bx)$" is false and "$\forall x(Ax \leftrightarrow Bx)$" is true?

Yes (No)

Is there a consistent set of 7 MPL WFFs such that each WFF in the set is inconsistent with "$\exists xBx$"?

(Yes) No

Is there an inconsistent set of 24 MPL WFFs such that each pair of WFFs in the set is consistent?

(Yes) No

Is there an SL WFF which contains no sentence letters other than "A" and "B", and which is entailed by every SL conjunction?

(Yes) No

Is there a consistent MPL WFF which is false under every interpretation?

Yes (No)

/18