Problem Set 1

Elementary Logic II

Due: 10 February 2010

Submit your problem set to Ms. Loletta Li in Main Building 312. Make sure your problem set is timestamped. Do not submit assignments by email. Late penalty: 20% for each day late. This problem set will not be accepted after 12 February.

Answer the questions on the problem set itself. Write neatly. If the grader cannot read your handwriting, you will not receive credit.

Be sure that all pages of the assignment are securely stapled together.

Check the course bulletin board for announcements about the assignment.

Do your own work.

If you copy your problem set, or permit others to copy, you may fail the course.

Name ______________________________

Student ID Number ______________________________

email ______________________________

Score: __________ of 100 marks

Due **10 February 2010** by **4:00PM**.

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1. (25 marks)

   True or false?
   Circle 'T' if the statement is true. Circle 'F' if the statement is false.
   For this question, you should assume that φ is a WFF of SL,
   and that "the system" refers to our SL natural deduction system.

   T F If a valid argument has all false premises, then its conclusion is false.
   T F The truth table method can always show whether or not an SL sequent is valid.
   T F "$A \rightarrow \sim B$" is an expression in SL.
   T F There is a derivation in the system of "$(A \rightarrow (\sim B \rightarrow \sim B))$" with no dependencies.
   T F "$A \& \sim A$" is an explicit contradiction in SL.
   T F Every correct derivation in the system has at least one line.
   T F Every correct derivation in the system uses at least two different rules.
   T F Any natural deduction system that is complete is also sound.
   T F Any natural deduction system that is sound is also complete.
   T F The conjunction of φ and the negation of φ is derivable with no dependencies in the system.

/25

2. (28 marks) Circle your answers.

a. Suppose our SL natural deduction system is revised by adding the following rule: *(NR1) If you have derived ψ, then you can write down $(\varphi \rightarrow \psi)$, depending on everything ψ depends on.*

Is the revised system sound? YES NO

Is the revised system complete? YES NO

b. Suppose our SL natural deduction system is revised by adding the following rule: *(NR2) You can write down $(\sim\varphi \vee \varphi)$ with no dependencies.*

Is the revised system sound? YES NO

Is the revised system complete? YES NO

c. Suppose our SL natural deduction system is revised by adding the following rule: *(NR3) If you have derived $(\varphi \rightarrow \psi)$, then you can write down $\sim(\varphi \& \psi)$, depending on everything $(\varphi \rightarrow \psi)$ depends on.*

Is the revised system sound? YES NO

Is the revised system complete? YES NO

d. Suppose rule PC is removed from our SL natural deduction system.

Is the revised system sound? YES NO

Is the revised system complete? YES NO

e. Suppose rule $\vee I$ is removed from our SL natural deduction system.

Is the revised system sound? YES NO

Is the revised system complete? YES NO

f. Suppose rule $\sim I$ is removed from our SL natural deduction system.

Is the revised system sound? YES NO

Is the revised system complete? YES NO

g. Suppose our SL natural deduction system is revised by adding the following rule: *(NR4) If you have derived $(\varphi \rightarrow \psi)$, and derived $(\psi \rightarrow \gamma)$,*
then you can write down $(\varphi \rightarrow \gamma)$, depending on everything $(\varphi \rightarrow \psi)$ depends on.

Is the revised system sound? YES NO

Is the revised system complete? YES NO

/28

3. (7 marks) For each of the following three attempted derivations, circle each line where a rule is misused.

1	1. $(A \vee B)$	A
2	2. B	A
2	3. $(B \vee C)$	2, $\vee$I
2	4. $((A \vee B) \rightarrow (B \vee C))$	1,3 $\rightarrow$I

1	1. $(B \& \sim B)$	A
2	2. A	A
2	3. $\sim A$	2,1 $\sim$I
	4. $(A \rightarrow \sim A)$	2,3 $\rightarrow$I

1	1. A	A
	2. $(A \rightarrow A)$	1,1 $\rightarrow$I
3	3. B	A
3	4. $(A \& B)$	2,3 &I
1,3	5. $((A \& B) \vee A)$	4,1 $\vee$I

/7

4. (40 marks) For the following, if it is possible, show it using the SL natural deduction system from this course. If it is not possible, write "not derivable".

(a) $(\sim C \vee (A \& B)) \vdash ((\sim C \vee A) \& (\sim C \vee B))$

1	1. $(\sim C \vee (A \& B))$	A
2	2. $\sim C$	A
3	3. $(A \& B)$	A
3	4. A	3 &E
3	5. B	3 &E
	6. $(\sim C \rightarrow \sim C)$	2,2 $\rightarrow$I
	7. $((A \& B) \rightarrow A)$	3,4 $\rightarrow$I
	8 $((A \& B) \rightarrow B)$	3,5 $\rightarrow$I
1	9. $(\sim C \vee A)$	1,6,7 PC
1	10. $(\sim C \vee B)$	1,6,8 PC
1	11. $((\sim C \vee A) \& (\sim C \vee B))$	9, 10 &I

(b) $\vdash ((A \& B) \leftrightarrow (B \& A))$

1	1. $(A \& B)$	A
1	2. A	1 &E
1	3 B	1 &E
1	4. $(B \& A)$	2,3 &I
	5. $((A \& B) \rightarrow (B \& A))$	1.4 $\rightarrow$I
6	6. $(B \& A)$	A
6	7. A	6 &E
6	8 B	6 &E
6	9. $(A \& B)$	7.8 &I
	10. $((B \& A) \rightarrow (A \& B))$	6.9. $\rightarrow$I
	11. $(((A \& B) \rightarrow (B \& A)) \& ((B \& A) \rightarrow (A \& B)))$	5,10 &I
	12 $((A \& B) \leftrightarrow (B \& A))$	11 $\leftrightarrow$I

(c) $\vdash ((A \rightarrow B) \rightarrow C) \rightarrow (A \rightarrow (B \rightarrow C))$

Not Derivable

(d) $((A \rightarrow B) \rightarrow C), \sim C \vdash \sim A$

Not Derivable

19,17 →I

18 (~(A→B) → (B→A))

19. ((A→B) ∨ (B→A)) 8,9,18 PC

(e) ⊢ (∼∼∼A ∨ ∼∼∼∼∼A)

1	1.	~(~~~A ∨ ~~~~~A)	A
2	2.	~~~A	A
2	3.	(~~~A ∨ ~~~~~A)	2∨I
1,2	4.	((~~~A ∨ ~~~~~A) & ~(~~~A ∨ ~~~~~A))	1,3&I
1	5.	~~~~A	2,4~I
1	6.	(~~~A ∨ ~~~~~A)	5∨I
	7.	((~~~A ∨ ~~~~~A) & ~(~~~A ∨ ~~~~~A))	1,6&I
	8.	(~~~A ∨ ~~~~~A)	1,7~E

(f) ⊢ ((A → B) ∨ (B → A))

1	1.	~((A→B) ∨ ~(A→B))	A
2	2.	(A→B)	A
2	3.	((A→B) ∨ ~(A→B))	2∨I
	4.	(((A→B ∨ ~(A→B)) & ((A→B) ∨ ~(A→B)))	1,3&I
1	5.	~(A→B)	2,4~I
1	6.	((A→B) ∨ ~(A→B))	5∨I
	7.	(((A→B) ∨ ~(A→B)) & ~((A→B) ∨ ~(A→B)))	1,6&I
	8	((A→B) ∨ ~(A→B))	1,7~E
	9.	((A→B) → (A→B))	2,2→I
10	10.	~(A→B)	A
11	11.	~A	A
12	12.	A	A
13	13.	B	A
~~12~~,13	14.	(A→B)	12,13 →I
10,13	~~...~~15.	((A→B) & ~(A→B))	10,14 &I
10,13	16	A	11,15~E
10	17	(B→A)	13,16 →I

(g) ⊢ ((A&C) → B)

Not Derivable

(h) (A ∨ B) ⊢ ∼∼(A ∨ B)

1	1.	(A∨B)	A
2	2.	~(A∨B)	A
1,2	3.	((A∨B) & ~(A∨B))	1,2&I
1	4.	~~(A∨B)	2,3 ~I

(i) (A ↔ B), ∼B, (A ∨ B) ⊢ (C → A)

1	1.	(A⇔B)	A
2	2.	~B	A
3	3.	(A∨B)	A
4	4.	C	A
2,3	5.	A	2,3∨E
1	6.	((A→B) & (B→A))	1⇔E
1	7.	(A→B)	6&E
1,2,3	8.	B	5,7 →E
1,2,3	9.	(B∨A)	8∨I
1,2,3	10.	A	2,9∨E
1,2,3	11.	((C→A)	4,10→I

(j) ∼(∼A& ∼ B) ⊢ (A ∨ B)

1	1.	~(~A&~B)	A
2	2.	~(A∨B)	A
3	3.	A	A
3	4.	(A∨B)	3∨I
2,3	5	((A∨B) & ~(A∨B))	2,4 &I
2	6.	~A	3,5 ~I
7	7.	B	A
7	8.	(A∨B)	7∨I
	9.	((A∨B) & ~(A∨B))	2,8&I
2,7			
2	10.	~B	7,9 ~I
2	11.	(~A&~B)	6,10& I
1,2	12.	((~A&~B) & ~(~A&~B)) /40	1,11&I
1	13.	(A∨B)	2,12~E