Problem Set 2

Elementary Logic II
Due: 14 April 2010

Submit your problem set to Ms. Loletta Li in Main Building 312. Make sure your problem set is timestamped. Do not submit assignments by email. Late penalty: 10% for each day late. This problem set will not be accepted after 16 April 2009.
Answer the questions on the problem set itself. Write neatly. If the grader cannot read your handwriting, you will not receive credit. Be sure that all pages of the assignment are securely stapled together. Check the course bulletin board for announcements about the assignment.
Do your own work. If you copy your problem set, or permit others to copy, you may fail the course.

Name Chun

Student ID Number ____________

Score: ____________

Due **14 April 2010** by **4:00PM.**

(1) (18 marks) *True or false? Circle 'T' if the statement is true. Circle 'F' if the statement is false. For this question, you should assume that φ is a WFF of MPL, and the derivations are in our natural deduction system for MPL.*

T F The truth table method can always determine whether or not an MPL formula is valid.
T F If an MPL sequent is valid, then our MPL natural deduction system can be used to show that the sequent is valid.
T F "$\exists x(Gx \leftrightarrow Gx)$" is a valid MPL formula.
T F There is an MPL derivation of "$\forall x(Gx \leftrightarrow Gx)$" with no dependencies.
T F Some natural deduction systems are neither sound nor complete.
T F If c is a constant and v is a variable, then $\varphi v/c$ is a well-formed formula of MPL.
T F No correct MPL derivation has more than 1000 lines.
T F Every natural deduction system that is complete is also sound.
T F If rule PC is removed from our MPL natural deduction system, then the resulting system would not be complete.
T F If φ is false under every interpretation then $\sim\varphi$ is valid.
T F The conjunction of φ and the negation of φ is derivable with no dependencies in our MPL natural deduction system.
T F If rule $\leftrightarrow I$ is removed, then the resulting system would not be sound.

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(2) (40 marks) For each of the following, show it using natural deduction, if it is possible. Otherwise write "Not possible".

(a) $\forall x(Fx\&Ga) \vdash (\forall xFx\&Ga)$

1	1	$\forall x(Fx \& Ga)$	A
1	2	$(Fb \& Ga)$	1 ∀E
1	3	Fb	2 &E
1	4	Ga	2 &E
1	5	$\forall x Fx$	3 ∀I
1	6	$(\forall x Fx \& Ga)$	4,5 &I

(b) $\forall x(Fx \to Ga) \vdash (\exists xFx \to Ga)$

1	1. $\forall x\,(Fx \to Ga)$	A
1	2. $(Fb \to Ga)$	1 $\forall$E
3	3. Fb	A
1,3	4. Ga	2,3 $\to$E
5	5. $\exists x\,Fx$	A
1,5	6. Ga	3,4,5 $\exists$E
1	7. $(\exists xFx \to Ga)$	5,6 $\to$I

(c) $\vdash (\sim\forall xAx \leftrightarrow \exists x \sim Ax)$

1	1. $\sim\forall xAx$	A
2	2. $\sim\exists x\sim Ax$	A
3	3. $\sim Aa$	A
3	4. $\exists x\sim Ax$	3 $\exists$I
2,3	5. $(\sim\exists x\sim Ax \;\&\; \exists x\sim Ax)$	2,4 &I
2	6. Aa	3,5 $\sim$E
2	7. $\forall xAx$	6 $\forall$I
1,2	8. $(\sim\forall xAx \;\&\; \forall xAx)$	1,7 &I
1	9. $\exists x\sim Ax$	2,8 $\sim$E
	10. $(\sim\forall xAx \to \exists x\sim Ax)$	1,9 $\to$I
11	11. $\exists x\sim Ax$	A
12	12. $\forall xAx$	A
13	13. $\sim Aa$	A
12	14. Aa	12 $\forall$E
12,13	15. $(\sim Aa \;\&\; Aa)$	13,14 &I
13	16. $\sim\forall xAx$	12,15 $\sim$I
11	17. $\sim\forall xAx$	11,13,16 $\exists$E
	18 $(\exists x\sim Ax \to \sim\forall xAx)$	11,17 $\to$I
	19. $(\sim\forall xAx \to \exists x\sim Ax) \;\&\; (\exists x\sim Ax \to \sim\forall xAx)$	10,18 &I
	20. $(\sim\forall xAx \leftrightarrow \exists x\sim Ax)$	19 $\leftrightarrow$I

(d) $(\forall xAx \vee \forall xBx) \vdash \forall x(Ax \vee Bx)$

1	1. $(\forall x\,Ax \vee Ax\;Bx)$	A
2	2. $\forall x\;Ax$	A
2	3. Aa	2 $\forall$E
	4. $(\forall xAx \to Aa)$	2,3 $\to$I
5	5. $\forall xBx$	A
5	6. Ba	5 $\forall$E
	7. $(\forall x\,Bx \to Ba)$	5,6 $\to$I
1	8. $(Aa \vee Ba)$	1,4,7 PC
1	9. $\forall x(Ax \vee Bx)$	8 $\forall$I

(e) $\exists x(Ax\&Bx) \vdash (\exists xAx \to \exists xBx)$

1	1. $\exists x\,(Ax \;\&\; Bx)$	A
2	2. $\exists x\;Ax$	A
3	3. $(Aa \;\&\; Ba)$	A
3	4. Ba	3 &E
3	5. $\exists x\;Bx$	4 $\exists$I
1	6. $\exists x\;Bx$	1,3,5 $\exists$E
1	7. $(\exists x\;Ax \to \exists x\;Bx)$	2,6 $\to$I

(f) $(Cm \vee \exists yCy) \vdash (Cm \vee \exists zCz)$

1	1. $(Cm \vee \exists y\,Cy)$	A
2	2. Cm	A
3	3. $\exists y\,Cy$	A
	4. $(Cm \to Cm)$	2 $\to$I
5	5. Ca	A
5	6. $\exists z\,Cz$	5 $\exists$I
3	7. $\exists z\,Cz$	3,5,6 $\exists$E
	8 $(\exists y\,Cy \to \exists z\,Cz)$	3,7 $\to$I
1	9. $(Cm \vee \exists z\,Cz)$	1,4,8 PC

(g) $(\exists x Ax \& \exists x Bx) \vdash \exists x(Ax \& Bx)$

Not Possible

(h) $(\exists x Ax \rightarrow \forall x(Bx \vee Cx)), \exists x((Ax \& \sim Bx) \& \sim Cx) \vdash \exists x Cx$

1	1.	$(\exists x\, Ax \rightarrow \forall x\, (Bx \vee Cx))$	A
2	2.	$\exists x\, ((Ax \& \sim Bx) \& \sim Cx)$	A
3	3.	$((Aa \& \sim Ba) \& \sim Ca)$	A
3	4.	$(Aa \& \sim Ba)$	3&E
3	5.	Aa	4&E
3	6.	$\sim Ba$	4&E
3	7.	$\exists x\, Ax$	5∃I
1,3	8.	$\forall x\, (Bx \vee Cx)$	1,7→E
1,3	9.	$(Ba \vee Ca)$	8∀E
1,3	10.	Ca	6,9∨E
1,3	11.	$\exists x\, Cx$	10∃I
1,2	12.	$\exists x\, Cx$	2,3,11∃E

(i) $(\exists x Bx \rightarrow \exists y Cy) \vdash \forall x(Bx \rightarrow \exists y Cy)$

1	1,	$(\exists x\, Bx \rightarrow \exists y\, Cy)$	A
2	2.	Ba	A
2	3.	$\exists x\, Bx$	2∃I
1,2	4.	$\exists y\, Cy$	1,3→E
1	5.	$(Ba \rightarrow \exists y\, Cy)$	2,4→I
1	6.	$\forall x\, (Bx \rightarrow \exists y\, Cy)$	5∀I

(j) $(\exists x Ax \vee \exists x Bx), \forall x((Ax \vee Bx) \rightarrow Cx) \vdash \exists x Cx$

1	1.	$(\exists x\, Ax \vee \exists x\, Bx)$	A
2	2.	$\forall x((Ax \vee Bx) \rightarrow Cx)$	A
3	3.	$\exists x\, Ax$	A
4	4.	Aa	A
4	5.	$(Aa \vee Ba)$	4∨I
2	6.	$((Aa \vee Ba) \rightarrow Ca)$	2∀E
2,4	7.	Ca	5,6→E
2,4	8.	$\exists x\, Cx$	7∃I
2,3	9.	$\exists x\, Cx$	3,4,8∃E
2	10.	$(\exists x\, Ax \rightarrow \exists x\, Cx)$	3,9→I
11	11.	$\exists x\, Bx$	A
12	12.	Bb	A
12	13.	$(Ab \vee Bb)$	12∨I
2	14.	$((Ab \vee Bb) \rightarrow Cb)$	2∀E
2,12	15.	Cb	13,14→E
2,12	16.	$\exists x\, Cx$	15∃I
2,11	17.	$\exists x\, Cx$	11,12,16∃E
2	18.	$(\exists x\, Bx \rightarrow \exists x\, Cx)$	11,17→I
1,2	19.	$(\exists x\, Cx \vee \exists x\, Cx)$	1,10,18PC
20	20.	$\sim \exists x\, Cx$	A
1,2,20	21.	$\exists x\, Cx$	19,20∨E
1,2,20	22.	$(\exists x\, Cx \& \sim \exists x\, Cx)$	20,21&I
1,2	23.	$\exists x\, Cx$	20,22~E

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(3) (25 marks) Circle your answer.

(a) Suppose the MPL natural deduction system is revised by adding the following rule:
(NR) for any variable v and constant c, if you have derived $\forall v(\varphi \rightarrow \psi)$, then you can write down $(\forall v\varphi \rightarrow \exists v\psi)$, depending on everything $\forall v(\varphi \rightarrow \psi)$ depends on.

Is the revised system sound? **YES** NO   Is the revised system complete? **YES** NO

(b) Suppose rule $\forall I$ is removed from the MPL natural deduction system.

Is the revised system sound? **YES** NO   Is the revised system complete? YES **NO**

(c) Suppose the MPL natural deduction system is revised by adding the following rule:
(NR1) for any variable v and constant c, if you have derived $\forall v(\varphi \vee (\psi \& \varphi))$, then you can write down $\varphi v/c$, depending on everything $\forall v(\varphi \vee (\psi \& \varphi))$ depends on.

Is the revised system sound? **YES** NO   Is the revised system complete? **YES** NO

(d) Suppose rule $\sim E$ is removed from the MPL natural deduction system.
Is the revised system sound? **YES** **NO** Is the revised system complete? **YES** **NO**
(e) Suppose the MPL natural deduction system is revised by adding the following rule:
(NR2) If you have derived $\forall\upsilon\varphi$, then you can write down $\exists\upsilon\varphi$, depending on everything $\forall\upsilon\varphi$ depends on.
Is the revised system sound? **YES** **NO** Is the revised system complete? **YES** **NO**

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(4) (10 marks) For each of the following four attempted derivations, circle each line where a rule is misused.

1	1. $(A\&B)$	A
2	2. $(A\&B)$	A
1	3. B	1, &E
2	4. A	2, &E
1,2	5. $(A\&B)$	3, 4 &I
2	6. $((A\&B) \to (A\&B))$	1, 5 $\to$I
	7. $(((A\&B) \to (A\&B)) \to (A\&B))$	2, 6 $\to$I

1	1. $(B \vee \sim B)$	A
2	2. B	A
1	3. $\sim B$	2,1 $\vee$E
1	4. $(B \vee \sim B)$	3 $\vee$I

1	1. $\forall x(Bx\&Aa)$	A
2	2. $(Ba\&Aa)$	1, $\forall$E
2	3. Ba	2 &E
1	4. $\forall xBx$	3 $\forall$I

1	1. $\exists y(\exists xAx \to By)$	A
2	2. Aa	A
3	3. $(\exists xAx \to Ba)$	A
2	4. $\exists xAx$	2 $\exists$I
2,3	5. Ba	3,4 $\to$E
2,3	6. $\exists xBx$	5 $\exists$I
1,2	7. $\exists xBx$	1,3,6 $\exists$E
1	8. $(Aa \to \exists xBx)$	2,7 $\to$I
1	9. $\forall y(Ay \to \exists xBx)$	1 $\forall$I

1 bonus mark for all correct

1.5 each correct circle

-1 for wrong circle

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(5) (7 marks)

Translate the following argument into MPL. Be sure to write down your translation scheme. Then either show that the argument is valid using natural deduction, or show that the argument is not valid by giving an appropriate interpretation.

Tennis players are fast. Only golfers are strong. Some golfers play tennis. Not all tennis players are golfers. Therefore some tennis players are not strong.

Tx: x is a tennis player
Gx: x is a golfer
Sx: x is strong
Fx: x is fast

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$\forall x(Tx \to Fx), \forall x(Sx \to Gx), \exists x(Gx \& Tx), \exists x(Tx \& \sim Gx) \models \exists x(Tx \& \sim Sx)$

1	1.	$\forall x\,(Tx \to Fx)$	A
2	2.	$\forall x\,(Sx \to Gx)$	A
3	3.	$\exists x\,(Gx \,\&\, Tx)$	A
4	4.	$\exists x\,(Tx \,\&\, \sim Gx)$	A
2	5.	$(Sa \to Ga)$	2 $\forall$I
6	6.	Sa	A
2.6	7.	Ga	5.6 $\to$E
8	8.	$(Ta \,\&\, \sim Ga)$	A
8	9.	Ta	8. &E
8	10.	$\sim Ga$	8. &E
2.6.8.	11.	$(Ga \,\&\, \sim Ga)$	7. 10. &I
2.8.	12.	$\sim Sa$	6. 11. $\sim$I
2.8.	13	$(Ta \,\&\, \sim Sa)$	9. 12. &I
2.8.	14.	$\exists x\,(Tx \,\&\, \sim Sx)$	13. $\exists$I
2.4	15.	$\exists x\,(Tx \,\&\, \sim Sx)$	4. 8. 14 $\exists$E
1,2,4.	16.	$(\exists x\,(Tx \,\&\, \sim Sx) \,\&\, \forall x\,(Tx \to Fx))$	1. 15 &I
1,2.4.	17.	$\exists x\,(Tx \,\&\, \sim Sx)$	16. &E
1,2,3,4.	18.	$(\exists x\,(Tx \,\&\, \sim Sx) \,\&\, \exists x\,(Gx \,\&\, Tx))$	3. 17. &I
1.2.3.4.	19.	$\exists x\,(Tx \,\&\, \sim Sx)$	18 &E