# Midterm Test
**Elementary Logic II**
2 March 2010

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Please write clearly.
You have 60 minutes to complete this test.

Student ID Number _____ *Chun* _____

Mark _____%

1. (30 marks)
   *True or false?*
   *Circle 'T' if the statement is true.*
   *Circle 'F' if the statement is false.*
   *"The system" is the natural deduction system for this course.*
   *Assume that $\varphi$ and $\psi$ are WFFs of SL.*

   T (F) If a natural deduction system for SL is not sound then it is complete.

   T (F) If the premises and conclusion of an argument are true, then the argument is valid.

   T (F) If $\varphi \vdash \psi$ is derivable in the system then $\psi$ is not a contradiction.

   (T) F $\vdash (\psi \rightarrow (\varphi \rightarrow \psi))$ is derivable in the system.

   T (F) If rule $\sim I$ is removed from the system, the resulting system would not be complete.

   T (F) If rule $\sim I$ is removed from the system, the resulting system would not be sound.

   (T) F Every correct derivation in the system uses the rule of assumption.

   T (F) There is no correct derivation in the system which uses every rule in the system.

   T (F) If rule PC is removed, then the resulting system would not be complete.

   (T) F The system cannot be used to show that $\varphi \models (\varphi \rightarrow \psi)$ is a valid sequent.

                                                                    /30

(2) (40 marks) If it is possible, show the following using the natural deduction system for this course. If it is not possible, write "not derivable".

   (a) $(S \& \sim R), ((P \rightarrow Q) \rightarrow R) \vdash \sim (P \rightarrow Q)$

   | 1 | 1. | $(S \& \sim R)$ | A |
   | 2 | 2. | $((P \rightarrow Q) \rightarrow R)$ | A |
   | 3 | 3. | $(P \rightarrow Q)$ | A |
   | 2,3 | 4. | $R$ | 2,3 $\rightarrow$ E |
   | 1 | 5. | $\sim R$ | 1 &E |
   | 1,2,3 | 6. | $(R \& \sim R)$ | 4,5 &I |
   | 1,2 | 7. | $\sim (P \rightarrow Q)$ | 3,6 $\sim$ I |

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(b) $P \vdash ((Q \to P) \to Q)$

Not Derivable

(not a valid sequue)

(c) $(A \& \sim B), (B \& \sim C) \vdash E$

| | | | |
|---|---|---|---|
| 1 | 1. | $(A \& \sim B)$ | A |
| 2 | 2. | $(B \& \sim C)$ | A |
| 3 | 3. | $\sim E$ | A |
| 1 | 4. | $\sim B$ | 1 &E |
| 2 | 5. | $B$ | 2 &E |
| 1.2 | 6. | $(B \& \sim B)$ | 4.5 &I |
| 1.2. | 7. | $E$ | 3.6. $\sim$E |

(d) $((A \lor B)\&C) \vdash ((A\&C) \lor (B\&C))$

| | | | |
|---|---|---|---|
| 1 | 1. | $((A \lor B) \& C)$ | A |
| 1 | 2. | $(A \lor B)$ | 1 &E |
| 1 | 3. | $C$ | 1 &E |
| 4 | 4. | $A$ | A |
| 1.4 | 5. | $(A \& C)$ | 3.4. &I |
| 1 | 6. | $(A \to (A \& C))$ | 4.5. →I |
| 7 | 7. | $B$ | A |
| 1.7 | 8. | $(B \& C)$ | 3.7 &I |
| 1 | 9. | $(B \to (B \& C))$ | 7.8. →I |
| 1 | 10. | $((A\&C) \lor (B\&C))$ | 2.6.9. P.C |

(e) $(\sim A \lor B), (\sim B \lor A) \vdash (A \leftrightarrow B)$

| | | | |
|---|---|---|---|
| 1 | 1. | $(\sim A \lor B)$ | A |
| 2 | 2. | $A$ | A |
| 3 | 3. | $\sim A$ | A |
| 2.3 | 4. | $(A \& \sim A)$ | 2.3 &I |
| 2 | 5. | $\sim \sim A$ | 3.4. ∼I |
| 1.2. | 6. | $B$ | 1.5. vE |
| 1 | 7. | $(A \to B)$ | 2.6. →I |
| 8 | 8. | $B$ | A |
| 9 | 9. | $\sim B$ | A |
| 8.9. | 10. | $(B \& \sim B)$ | 8.9. &I |
| 8 | 11. | $\sim \sim B$ | 9.10. ∼I |
| 12 | 12. | $(\sim B \lor A)$ | A |
| 8.12. | 13. | $A$ | 11.12. vE |
| 12 | 14. | $(B \to A)$ | 8.13. →I |
| 1.12. | 15. | $((A \to B) \& (B \to A))$ | 7.14. &I |
| 1.12. | 16. | $(A \leftrightarrow B)$ | 15 ⟷I |

40

(3) (24 marks) Circle your answer.

  a. Suppose our natural deduction system is revised by adding the following rule:
(NR) if you have derived $\psi$, then you can write down $(\varphi \rightarrow \psi)$, depending on everything $\psi$ depends on.

Is the revised system sound?     **YES**         **NO**

Is the revised system complete?    **YES**         **NO**

  b. Suppose that rule $\sim$E and rule $\sim$I are both removed from our natural deduction system.

Is the revised system sound?     **YES**         **NO**

Is the revised system complete?    **YES**         **NO**

  c. Suppose our natural deduction system is revised by adding the following rule:
(NR1) if you have derived $\varphi$ and you have derived $\psi$ then you can write down $(\varphi \vee \psi)$, depending on everything $\psi$ depends on.

Is the revised system sound?     **YES**         **NO**

Is the revised system complete?    **YES**         **NO**

  d. Suppose our natural deduction system is revised by adding the following rule:
(NR2) if you have derived $\varphi$ and you have derived $\psi$ then you can write down $(\varphi \rightarrow \psi)$ depending on everything $\varphi$ depends on.

Is the revised system sound?     **YES**         **NO**

Is the revised system complete?    **YES**         **NO**

                                             /24

(4) (6 marks) Circle each line where a rule is misused.

| 1 | 1. $A$ | A |
| 1 | 2. $(A\&A)$ | 1,1 &I |
| 1 | 3. $((A\&A) \vee (A\&A))$ | 2 $\vee$I |

| 1 | 1. $(B \rightarrow \sim A)$ | A |
| 2 | 2. $B$ | A |
| 2 | 3. $\sim A$ | 2,1 $\rightarrow$E |
| 2 | 4. $(\sim A \leftrightarrow B)$ | 2,3 $\leftrightarrow$I |

| 1 | 1. $A$ | A |
| 2 | 2. $B$ | A |
| 2 | 3. $(A \rightarrow B)$ | 1,2 $\rightarrow$I |
| 1,2 | 4. $B$ | 1,3 $\rightarrow$E |
| | 5. $((A \rightarrow B) \rightarrow B)$ | 3,4 $\rightarrow$I |

                                             /6

<div align="center">END OF PAPER</div>