Problem Set 2
Elementary Logic II
Due: 14 April 2010

Submit your problem set to Ms. Loletta Li in Main Building 312. Make sure your problem set is timestamped. Do not submit assignments by email. Late penalty: 10% for each day late. This problem set will not be accepted after 16 April 2009.

Answer the questions on the problem set itself. Write neatly. If the grader cannot read your handwriting, you will not receive credit. Be sure that all pages of the assignment are securely stapled together. Check the course bulletin board for announcements about the assignment.

Do your own work. If you copy your problem set, or permit others to copy, you may fail the course.

Name _____

Student ID Number _____

Score: _____

Due **14 April 2010** by **4:00PM**.

(1) (18 marks) *True or false? Circle 'T' if the statement is true. Circle 'F' if the statement is false. For this question, you should assume that φ is a WFF of MPL, and the derivations are in our natural deduction system for MPL.*

    T   F   The truth table method can always determine whether or not an MPL formula is valid.

    T   F   If an MPL sequent is valid, then our MPL natural deduction
                   system can be used to show that the sequent is valid.

    T   F   "$\exists x(Gx \leftrightarrow Gx)$" is a valid MPL formula.

    T   F   There is an MPL derivation of "$\forall x(Gx \leftrightarrow Gx)$" with no dependencies.

    T   F   Some natural deduction systems are neither sound nor complete.

    T   F   If c is a constant and v is a variable, then $\varphi v/c$ is a well-formed formula of MPL.

    T   F   No correct MPL derivation has more than 1000 lines.

    T   F   Every natural deduction system that is complete is also sound.

    T   F   If rule PC is removed from our MPL natural deduction system,
                   then the resulting system would not be complete.

    T   F   If φ is false under every interpretation then $\sim\varphi$ is valid.

    T   F   The conjunction of φ and the negation of φ is derivable
                   with no dependencies in our MPL natural deduction system.

    T   F   If rule $\leftrightarrow I$ is removed, then the resulting system would not be sound.

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(2) (40 marks) For each of the following, show it using natural deduction, if it is possible. Otherwise write "Not possible".

    (a) $\forall x(Fx \& Ga) \vdash (\forall x Fx \& Ga)$

(b) $\forall x(Fx \rightarrow Ga) \vdash (\exists x Fx \rightarrow Ga)$

(c) $\vdash (\sim\forall x Ax \leftrightarrow \exists x \sim Ax)$

(d) $(\forall x Ax \lor \forall x Bx) \vdash \forall x(Ax \lor Bx)$

(e) $\exists x(Ax \& Bx) \vdash (\exists x Ax \rightarrow \exists x Bx)$

(f) $(Cm \lor \exists y Cy) \vdash (Cm \lor \exists z Cz)$

(g) $(\exists xAx \& \exists xBx) \vdash \exists x(Ax \& Bx)$

(h) $(\exists xAx \to \forall x(Bx \lor Cx)), \exists x((Ax \& \sim Bx) \& \sim Cx) \vdash \exists xCx$

(i) $(\exists xBx \to \exists yCy) \vdash \forall x(Bx \to \exists yCy)$

(j) $(\exists xAx \lor \exists xBx), \forall x((Ax \lor Bx) \to Cx) \vdash \exists xCx$

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(3) (25 marks) Circle your answer.

(a) Suppose the MPL natural deduction system is revised by adding the following rule:
(NR) for any variable v and constant c, if you have derived $\forall v(\varphi \to \psi)$, then you can write down $(\forall v\varphi \to \exists v\psi)$, depending on everything $\forall v(\varphi \to \psi)$ depends on.

Is the revised system sound? **YES NO**　　Is the revised system complete? **YES NO**

(b) Suppose rule $\forall I$ is removed from the MPL natural deduction system.

Is the revised system sound? **YES NO**　　Is the revised system complete? **YES NO**

(c) Suppose the MPL natural deduction system is revised by adding the following rule:
(NR1) for any variable v and constant c, if you have derived $\forall v(\varphi \lor (\psi \& \varphi))$, then you can write down $\varphi v/c$, depending on everything $\forall v(\varphi \lor (\psi \& \varphi))$ depends on.

Is the revised system sound? **YES NO**　　Is the revised system complete? **YES NO**

(d) Suppose rule $\sim E$ is removed from the MPL natural deduction system.

Is the revised system sound? **YES NO**   Is the revised system complete? **YES NO**

(e) Suppose the MPL natural deduction system is revised by adding the following rule:
(NR2) If you have derived $\forall v\varphi$, then you can write down $\exists v\varphi$, depending on everything $\forall v\varphi$ depends on.

Is the revised system sound? **YES NO**   Is the revised system complete? **YES NO**

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(4) (10 marks) For each of the following four attempted derivations, circle each line where a rule is misused.

1	1. $(A\&B)$	A
2	2. $(A\&B)$	A
1	3. B	1, &E
2	4. A	2, &E
1,2	5. $(A\&B)$	3, 4 &I
2	6. $((A\&B) \to (A\&B))$	1, 5 →I
	7. $(((A\&B) \to (A\&B)) \to (A\&B))$	2, 6 →I

1	1. $(B \vee \sim B)$	A
2	2. B	A
1	3. $\sim B$	2,1 ∨E
1	4. $(B \vee \sim B)$	3 ∨I

1	1. $\forall x(Bx\&Aa)$	A
2	2. $(Ba\&Aa)$	1, ∀E
2	3. Ba	2 &E
1	4. $\forall xBx$	3 ∀I

1	1. $\exists y(\exists xAx \to By)$	A
2	2. Aa	A
3	3. $(\exists xAx \to Ba)$	A
2	4. $\exists xAx$	2 ∃I
2,3	5. Ba	3,4 →E
2,3	6. $\exists xBx$	5 ∃I
1,2	7. $\exists xBx$	1,3,6 ∃E
1	8. $(Aa \to \exists xBx)$	2,7 →I
1	9. $\forall y(Ay \to \exists xBx)$	1 ∀I

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(5) (7 marks)

Translate the following argument into MPL. Be sure to write down your translation scheme. Then either show that the argument is valid using natural deduction, or show that the argument is not valid by giving an appropriate interpretation.

Tennis players are fast. Only golfers are strong. Some golfers play tennis. Not all tennis players are golfers. Therefore some tennis players are not strong.

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