Argument for CSL-Equivalence given the Classical Assumption

(CSL-Equivalence) For any sentence A in S, and for any set Σ of sentences in S,

A is a CSL-semantic consequence of the sentences in Σ iff A is a logical consequence of the sentences in Σ

(Classical Assumption) Every meaning (appropriate for a sentence) is either true or false

In order to establish CSL-Equivalence, I will first establish four preliminary lemmas.

Lemma 1 For any meaning interpretation m, there is a CSL-interpretation v such that

A is true under v iff A is true under m

Argument for Lemma 1. The argument for Lemma 1 is by induction on the length of sentences in S

A ~~rgument for lemma~~.

~~Base case~~: ~~By definition~~, Let m be a meaning interpretation of S. Let v be the CSL-interpretation such that, for any sentential constant p in S,

$$v(p) = 1 \quad \text{iff} \quad m(p) \text{ is true}.$$

Base case: By definition, for each sentential constant p, p is true under v iff p is true under m

The case of negation

$\neg A$ is true under v_m

iff A is not true under v_m

(By definition of how CSL-interpretations are extended to apply to all sentences)

iff A is not true under m

(By the inductive assumption that A satisfies Lemma 1)

iff $\neg A$ is true under m

(Since '$\neg$' means 'it is not the case that' and given the classical assumption every sentence meaning is either true or false)

Given the case of disjunction is similarly proven, this establishes lemma 1.

Since the argument for lemma 1 does not depend on any contingent facts, it follows that Lemma 1 is not only true, but

Inductive case: We need to prove that, if sentences A and B satisfy Lemma 1 then $\neg A$, $(A \wedge B)$, and $(A \vee B)$, ~~$(A \supset B)$ and $(A \equiv B)$~~ do as well

I will do the case of conjunction and ~~the~~ the case of ~~disju~~ negation. The case of disjunction ~~other cases are~~ is similar.

The Case of Conjunction

$(A \wedge B)$ is true under v_m

iff A is true under v_m and B is true under v_m (By the definition of how CSL-interpretations are extended to apply to all sentences)

iff A is true under m and B is true under m (By the inductive assumption that A and B satisfy lemma 1)

iff $(A \wedge B)$ is true under m (since '$\wedge$' means 'and' ~~By the definition of how meaning interpretations are extended to apply to all sentences~~)

~~Given the other cases are similarly proven, this establishes lemma 1~~

necessarily true. In particular, we have:

Lemma 2 For any meaning interpretation m, necessarily, there is a CSL-interpretation v such that

A is true under v iff A is true under m

Lemma 3 For any CSL-interpretation v, there is a meaning interpretation m such that, for any sentence A in S,

A is true under m iff A is true under v

Argument for Lemma 3. Let v be a CSL-interpretation. Let m be a meaning interpretation such that, for any sentential constant p, if $v(p)=1$ then $m(p)$ is true, while if $v(p)=0$ then $m(p)$ is false.

The rest of the argument is similar to the argument for lemma 1. □

Lemma 4 For any CSL-interpretation v,
for any sentence A in S,
A is true under v iff, necessarily, A is true under v

Argument for Lemma 4. Sentences in A and CSL-interpretations are abstract objects where nature is not contingent. Hence the left hand side entails the right hand side. Since what is necessary is true, the right hand side also entails the left hand side.

Def 2 For any CSL-interpretation v, let m_v be ~~the~~ a meaning interpretation such that, for any sentential constant p, if $v(p)=1$ then $m_v(p)$ is ~~[illegible]~~ be some true proposition, while if $v(p)=0$ then $m_v(p)$ is ~~[illegible]~~ some ~~[illegible]~~ ~~no~~ ~~[illegible]~~ false proposition.

Lemma 2 For any sentence A in S~~[illegible]~~,

A is true under m_v iff A is true under v

Proof of Lemma 2. Similar to the proof of Lemma 1.

Proof of (CSL-Equivalence). I will first do the left to right direction, before doing the ~~The~~ right to left direction. ~~is similar~~.

Suppose A is not a CSL-semantic consequence of the sentences in Σ

Then (~~[illegible]~~) there is a CSL-interpretation v such that all the sentences in Σ are true under v, and A is not true under ~~[illegible]~~ v.

→

Then (by Lemma 3) there is a meaning interpretation m_v such that all sentences in Σ are true under m_v, and A is not true under m_v.

Hence there is a meaning interpretation under which ~~Hence~~ A is ~~(logically [illegible]~~ not a metaphysical consequence of the sentences in Σ.

Hence, ~~logical consequence~~) A is not a logical consequence of the sentences in Σ

Now for the right to left direction.

Suppose A is not a logical consequence of the sentences in Σ.

Then there is a meaning interpretation m ~~such that under which~~ such that it is possible that all the sentences in Σ are true under ~~Σ~~ m, but A is false under m.

Hence, by Lemma 2, it is possible that there is a CSL-int v such that all the sentences in Σ are true under v, but A is false under v.

Hence by Lemma 4, there is a CSL-int v such that all the sentences in Σ are true under v, but A is false under v

Hence A is not a CSL-semantic conseq of the sentences in Σ

This establishes the right to left direction.