What is Logic?

Seminar 1

PHIL2520 Philosophy of Logic

18 September 2012

Reading for this seminar

Required reading:

Sider's 'What is Logic?' pp. 1-11

(See course website)

Optional reading:

Macfarlane's 'Logical Constants'

See http://plato.stanford.edu/entries/logical-constants/

What is logic?

The central aim of logic is to give an account of logical consequence.

In particular, logic aims to give an account of:

i) What it is for a sentence to be a logical consequence of some other sentences, and

ii) Which sentences are logical consequences of which other sentences

A closely related aim

Logicians are also interested in the closely related notion of validity.

These notions are closely related since:

A valid argument is an argument whose conclusion is a logical consequence of its premises

Examples of arguments

1. Either it is not the case that John is a man or John is mortal
   John is a man

   John is mortal

2. Either it is not the case that John is a man or John is mortal
   Ruth is a woman

   John is mortal

Examples of arguments (cont)

3. John is a bachelor

   John is unmarried

4. The sea contains water

   The sea contains H_2O

What do logicians mean by ‘logical consequence’?

Logicians typically use `Q is a logical consequence of P1,…, Pn’ to mean `Q follows from P1,…,Pn in virtue of their **logical form**’

But what does this mean?

Logical expressions

An attractive account appeals to the distinction between logical and non-logical expressions.

Paradigm examples of logical expressions: `and', `or', `not', 'some', `all', 'if…then'

Paradigm examples of non-logical expressions: 'John', 'Ruth', 'is a man', 'is mortal', 'is a woman', 'is a bachelor', 'is unmarried', 'the sea', 'water', 'H2O'

A Quinean account of logical consequence

Def: A meaning interpretation assigned semantically appropriate meanings to expressions. (For example, predicates get assigned properties.)

The Quinean account: Q is a logical consequence of P1,…,Pn iff, for any meaning interpretation m of the simple non-logical expressions in P1,…,Pn, and Q, if P1,…, Pn are all true under m then Q is true under m

The Quinean account and the examples

Given the Quinean account, argument 1 is valid, while arguments 2-4 are not valid.

What is a logical expression?

The topic neutral account: logical expressions are completely general, while non-logical expressions are about a particular subject matter

See MacFarlane's 'Logical Constants' for discussion of this and other accounts

Logical truth

A is a logical truth iff A is true in virtue of its logical form

Paradigm examples of logical truths:

i) Snow is white or it is not the case that snow is white
ii) If snow is white and snow is white

Formal languages

Due to the complexity of natural languages such as English, it is hard to apply the above account of logical consequence to natural language sentences.

Instead, logicians study logical consequence in simpler formal languages

The language of sentential logic (SL)

The symbols of SL are:

i) Sentence constants (Priest calls them ‘propositional parameters’) p0, p1, p2,…

ii) ‘~’ meaning ‘it is not the case that’

iii) ‘&’ meaning ‘and’

iv) ‘v’ meaning ‘or’

v) ‘$\supset$’, where ‘(A$\supset$B)’ means the same as ‘(~A v B)’

vi) ‘≡’, where ‘(A≡B)’ means the same as ‘((A$\supset$B)v (B$\supset$A))’

The language of sentential logic (cont)

The sentences of SL are:

i) This sentential constants p0, p1, p2,…
ii) The strings of symbols that can be generated by the following rule:

If A and B are formulas in SL then ~A, (A & B), (A v B), (A⊃B), (A≡B) are sentences in SL

Meaning interpretations of SL

A meaning interpretation m of SL is a function that maps each sentence constant p to a proposition

Meaning interpretations of SL (cont)

If m is an interpretation of SL, then extend m to all sentences of SL so that

i) m(~A) = the negation of m(A)

ii) m(A&B) = the conjunction of m(A) and m(A)

iii) m(A v B) = the disjunction of m(A) and m(A)

iv) m(A⊃B) = m(~A v B)

v) m(A≡B) = m((A⊃B)&(B⊃A))

Why this way of extending m is correct

If the sentential constants have the meanings assigned by a meaning interpretation m, then any sentence A will have the meaning assigned by m's extension defined above

Example: Suppose p0 expresses the proposition that John is a man, and p1 expresses the proposition that Ruth is a woman, and these are the propositions assigned to them by m. Then (p0 & p1) expresses the proposition that John is a man and Ruth is a woman, which is the meaning assigned to it by the extension of m

The Quinean account of logical consequence applied to SL

Def: A is true under m iff m maps A to a true proposition

Let A be a sentence in SL, and Σ be a set of sentences in SL.

A is a logical consequence of the sentences in Σ iff, for any meaning interpretation m of SL, if all the members of Σ are true under m then A is true under m

SL-interpretations

In order to study at logical consequence in the language SL, logicians don’t employ many interpretations.

Instead, they employ simpler SL-interpretations, where:

A SL-interpretation v of SL is a function that maps each sentence constant either 1 or 0

SL-interpretations (cont)

If v is an SL-interpretation of SL, then extend v to all sentences of SL so that it maps all sentences to either 1 or 0 in such a way that:

i) $v(\sim A)=1$ iff $v(A)=0$

ii) $v(A \& B)=1$ iff $v(A)=1$ and $v(B)=1$

iii) $v(A \vee B)=1$ iff either $v(A)=1$ or $v(B)=1$

iv) $v(A \supset B)=1$ iff either $v(A)=0$ or $v(B)=1$

v) $v(A \equiv B)=1$ iff $v(A)=v(B)$

SL-semantic consequence

Def: A is true under a SL-interpretation v iff v(A)=1

Let A be a sentence in SL, and Σ be a set of sentences in SL.

Def: A is a SL-semantic consequence of the sentences in Σ iff, for any SL-interpretation v of SL, if all the members of Σ are true under v then A is true under v

An equivalence

Given the Quinean account of logical consequence, and given plausible assumptions, it can be shown that:

(Equivalence) For any sentence A in SL, and any set of sentences Σ in SL,

A is a logical consequence of the entences in Σ iff A is a SL-semantic consequence of the sentences in Σ

The argument for this will put up on the course webpage

(Challenging) Excercise

An alternative to the Quinean account of logical consequence is the following:

Q is a logical consequence of P1,…,Pn iff, for any meaning interpretation m of the simple non-logical expressions in P1,…,Pn, and Q, **necessarily**, if P1,…, Pn are all true under m then Q is true under m

Exercise: Argue that (Equivalence) is still true given this alternative account of logical consequence