Many Valued Logics

Seminar 10

PHIL2520 Philosophy of Logic

11 December 2012

Administration

Reading for today: Priest Ch 7

Wed, 12 Dec, 3.30-4.30pm, philosophy seminar room: Solution session for assignment 3

18 Dec: Exam 1.30pm-3.30pm CPD 2.14

18 Dec: Exam 6pm-8pm Philosophy seminar room (for students who can't make earlier exam time due to clashes. You need to contact me if you want to attend this exam.)

8 Jan: Essay Due 5pm Email submission to danm@hku.hk

The ceteris paribus account

The above problem motivates the following theory.

The ceteris paribus account: `If A, B’ is true iff ‘$\Box((A \,\&\, C_A) \supset B)$’ is true,
where ‘C_A’ roughly means ‘everything other than A remains the same’).

Note: ‘Ceteris Paribus’ is latin for ‘other things being equal’

The ceteris paribus account (cont)

The account formulated in terms of worlds:

‘If A, B’ is true at world u iff, for any world v at which A is true, and at which everything other than A remains the same as in u, B is true at v

What does ‘everything (other than A) remains the same at v as in u’?

Simple but bad answer: ‘everything (other than A) remains the same at v as in u’ is true iff, for any proposition p (other than the proposition expressed by A), p is true at v iff p is true at u.

Argument against: Suppose A is false at u, but true at v, and C is true at both u and v. Then ‘A&C’ is false at u, but true at v. Hence, on the the simple answer, there is no world v (other than u) such that “everthing (other than A) remains the same at v as at u”. But this would make the ceteris paribus account implausible.

A better answer

The answer: Everthing (other than A) remains the same at v as at u iff v is the most similar world to u at which A is true.

Robert Stalnaker a sophisticated version of the ceteris paribus acount, understood with this reading of ‘Everthing (other than A) remains the same at v as at u’. See ‘Indicative Conditionals’.

Problems for the ceteris paribus account

The ceteris paribus account still suffers some of the problems faced by the strict conditional account.

For example, the account still entails (PSI1) and (PSI2).

(PMI1) For any sentences A and B, if '◊A' is false, then `If A, B' is true

(PMI2) For any sentences A and B, if '□B' is true, then `If A, B' is true

Bivalency

The logics we have been studying all assume:

(Bivalency) For any (meaningful) sentence S, S is either true or false, but not both.

Some philosophers (such Graham Priest) have rejected (Bivalency), and instead endorsed either (Gluts) or (Gaps) or both.

(Gluts) Some sentences are both true and false

(Gaps) Some sentences are neither true nor false.

Case 1: Inconsistent laws

Suppose the constitution in some country X contains the following clauses:

(1) No aborigine shall have the right to vote

(2) All property holders shall have the right to vote.

Suppose John is an aborigine who comes to own some property. Then John both has the right to vote, and doesn't have the right to vote. So 'John has the right to vote' is both true and false.

Case 2: The liar paradox

(3) This sentence is false

Suppose (3) is true. Then (3) is false, and hence both true and false

Suppose (3) is false. The (3) is true, and hence both true and false.

Hence, either way, (3) is both true and false.

Response to case 2

The above argument relies on the assumption that (3) is either true or false.

But this assumption is incorrect, since (3) is defective, and hence is neither true nor false.

If this response is correct then we have a truth value gap, rather than a truth value glut.

Case 3: Denotation failure

According to Frege, certain sentences containing non-referring expressions are neither true nor false.

Examples:

(4) The biggest integer is even

(5) The present king of France is bald

Response to case 3

Not everyone agrees with Frege.

Russell thinks that (4) and (5) are both false, since their meanings are given by (4’) and (5’) respectively.

(4’) There is one and only one biggest integer, and every biggest integer is even

(5’) There is one and only one present king of France, and every present king of France is bald

Other cases

Other cases of alleges truth value gluts and gaps include:

i) Russell's paradox

ii) Fiction names

iii) Future contingents

See Priest p130-133

Towards many valued logic

Let C = {&,v,~, ⊃, ≡}. Classical sentential logic can be thought of as being the triple <V, D, {f_c|c∈C}>, where

i) V is the set of truth values {1,0}

ii) D is the set of designated values {1}

iii) For each connective c in C, f_c is the truth function c corresponds to.

Ex: $f_{\sim}(1)=0$, $f_{\sim}(0)=1$

[Discuss the other truth functions–See Priest p 121]

Towards many valued logic (cont)

An interpretation v is a function assigning each sentential constant to a member of V={1,0}.

An interpretation v is extended to apply to all sentences in S* by applying the appropriate truth functions recursively.

Ex: $v[\sim(p \vee q)] = f_{\sim}(v[p \vee q]) = f_{\sim}(f_{\vee}(v[p],v[q]))$

Q is a logic $\langle V, D, \{f_c \mid c \in C\} \rangle$ semantic consequence of P1,…,Pn iff there is no interpretation v that assigns each P1,…Pn to a member of D={1}, but does not assign Q to a member of D={1}.

Many valued logics for S*: The general structure

A many valued logic for the language S* is a triple $<V, D, \{f_c | c \in C\}>$, where

i) V is a set of truth values

ii) D is a subset of V

iii) For each connective c in C, f_c is the truth function c corresponds to.

Many valued logics for S* (cont)

An interpretation v for $<V, D, \{f_c|c\in C\}>$ is a function assigning each sentential constant to a member of V.

An interpretation v is extended to apply to all sentences in S* by applying the appropriate truth functions recursively.

Q is a $<V, D, \{f_c|c\in C\}>$ semantic consequence of P1,…,Pn iff there is no interpretation v that assigns each P1,…Pn to a member of D, but does not assign Q to a member of D.

Kleene’s three valued logic (K3)

K3=<V, D, $\{f_c | c \in C\}$>, where

i) V = {1,0,i} (thought of as **true, false**, and **neither true nor false**)

ii) D = {1}

iii) The truth functions are as on p 122 Priest.

[Discuss motivation and example in 7.3.5]

Semantic truths in K3

The law of excluded middle is not a semantic truth in K3 (that is, ‘pv~p’ is not assigned a value in D by all interpretations)

In fact there are no semantic truths in K3 (See Priest 7.14 problem 3)

The three valued logic LP

LP=<V, D, $\{f_c | c \in C\}$>, where

i) V = {1,0,i} (thought of as **only true**, **only false**, and **both true and false**)

ii) D = {1,i}

iii) Same as for K3

[Discuss motivation for these truth functions. See Priest p 124]

Semantic truth and consequence in LP

(6) ‘p v ~p’ is a LP semantic truth

(7) q is not an LP semantic consequence of ‘p & ~p’

(8) q is not an LP semantic consequence of p and ‘p ⊃ q’