# Classical Sentential Logic

Seminar 2

PHIL2520 Philosophy of Logic

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# Reading

Required reading for this seminar:

Priest, Ch 1, Sec 1.1-1.5, 1.11

Required reading for next seminar (note change):

Priest, Ch 1, Sec 1.6-1.10

Optional reading for next seminar:

Jackson, 'On assertion and indicative conditionals'

(Jackson's paper presupposes basic knowledge of probability theory. If you are unfamiliar with probability theory, a good introduction is Skyrms, Ch 6)

See course webpage

# What is logic?

Logic is primarily concerned with studying arguments.

Examples of questions addressed by logicians: which arguments are good arguments, what types of good arguments are there?

# An important distinction among arguments

Def: An argument is **metaphysically valid** iff, necessarily, if its premises are true then its conclusion is true

# Another important distinction among arguments

Informal Def 1: An argument is **logically valid** iff it is metaphysically valid ``in virtue of its logical form’’

Informal Def 2: An argument is **logically valid** iff one can tell that it is metaphysically valid even if one does not know what it is non-logical expressions mean

# Meaning interpretations

A more precise definition can be given using the notion of a meaning interpretation.

Def: A **meaning interpretation** is a mapping which assigns each non-logical expression meaning appropriate for its syntactic type

At least initially plausible claims (which I will assume for now):

i) The meaning of a name such as `Obama' is the object it refers to
ii) The meaning of a predicate such as `is happy' is the property it expresses
iii) The meaning of a sentence is the proposition it expresses

# Meaning interpretations (cont)

Given these claims, a meaning interpretation assigns:

i) each name to an object
ii) each predicate to a property, and
iii) each sentence to a proposition

Example: Let m be the meaning interpretation which assigns `Obama’ to Brad Pitt, and `is a bachelor’ to the property of being an actor. Then, under m, `Obama is a bachelor’ means Brad Pitt is an actor.

# Meaning interpretations (cont)

Def: A sentence is true under a meaning interpretation m iff the sentence is true given its non-logical expressions have the meanings assigned by m

Example continued: Under the meaning interpretation m which assigns `Obama' to Brad Pitt, and `is a bachelor' to the property of being an actor, `Obama is a bachelor' is true.

# A precise definition of logical validity

Preliminary Def: An argument is **metaphysically valid under a meaning interpretation m** iff, necessarily, if its premises are true under m then its conclusion is true under m

Precise Def: An argument is **logically valid** iff, for any meaning interpretation m, the argument is metaphysically valid under m

# Examples of arguments

1. Either it is not the case that John is a man or John is mortal

   John is a man

   ---------------------------------------------------------------------------------

   John is mortal

2. Either it is not the case that John is a man or John is mortal

   Ruth is a woman

   ---------------------------------------------------------------------------------

   John is mortal

# Examples of arguments (cont)

3. John is a bachelor

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John is unmarried

# Metaphysical and logical consequence

Def: Q is a **metaphysical consequence** of P1,…, Pn iff, necessarily, if P1,…, Pn are all true, then Q is true

Def: Q is a **logical consequence** of P1,…,Pn iff, for any meaning interpretation m, Q is a metaphysical consequence of P1,…,Pn under m

Excercise: How does this definition of logical consequence differ from the Quinean definition given in seminar 1? (I will take the above as the offical definition of logical consequence.)

# Metaphysical and logical truth

Def: A sentence is a **metaphysical truth** iff it is necessarily true

Def: A sentence is a **logical truth** iff, from any meaning interpretation m, it is metaphysically true under m

# The language S

Due to the complexity of natural languages such as English, logicians study logical validity and logical consequence in simpler formal languages, such as S.

The symbols of S are:

i) Sentence constants (Priest calls them 'propositional parameters') p0, p1, p2,…
ii) '~' meaning 'it is not the case that'
iii) '&' meaning 'and'
iv) 'v' meaning 'or'

# The language S (cont)

The sentences of S are:

i) This sentential constants p0, p1, p2,…

ii) The strings of symbols that can be generated by the following rule:

If A and B are formulas in S then ~A, (A & B), (A v B) are sentences in S.

A meaning interpretation m of S is a function that maps each sentence constant p to a proposition

# CSL-interpretations

In order to study logical consequence in the language S, logicians don't employ meaning interpretations.

Instead, the classical approach is to employ simpler CSL-interpretations.

Def: A **CSL-interpretation** m of S is a function that maps each sentence constant in S to either 1 (representing truth) or 0 (representing falsehood)

# CSL-interpretations (cont)

If m is an CSL-interpretation of S, v is extended to all sentences of S so that it maps all sentences to either 1 or 0 in such a way that:

i) v(~A)=1 iff v(A)=0

ii) v(A&B)=1 iff v(A)=1 and v(B)=1

iii) v(A v B)=1 iff either v(A)=1 or v(B)=1

# CSL-semantic consequence

Def: A is true under a CSL-interpretation v iff v(A)=1

Let A be a sentence in S, and Σ be a set of sentences in S.

Def: A is a **CSL-semantic consequence** of the sentences in Σ ($\Sigma \models_{CSL} A$) iff, for any CSL-interpretation v of S, if all the members of Σ are true under v then A is true under v

# Logical consequence and CSL-semantic consequence

Classical Assumption: Every meaning (appropriate for a sentence) is either true or false (and not both!)

(CSL-Equivalence) A is a CSL-semantic consequence of the sentences in Σ ($\Sigma \mid=_{CSL} A$) iff A is a logical consequence of the sentences in Σ

Rough argument: In S, the truth values of complex sentences is determined by the truth values of its sentence constants. Hence meaning interpretations can be replaced by CSL-interpretations.

Rigorous argument: See course webpage

# Objection

Some sentences may have meanings under which they are neither true nor false.

An alternative approach that allows for sentences to fail to either true or false will be discussed later in the course.

For now I will assume the classical assumption is correct.

# CSL-Tableaux

It would be good to have a way of proving that A is a CSL-semantic consequence of Σ, or that A is not a CSL-semantic consequence of Σ.

CSL-Tableaux proofs provide a way of doing this (see Priest, sec 1.4 and 1.5).

Definitions: Tableaux, rules of generating CSL Tableaux, branches, complete CSL Tableaux, closed/open branches, closed/open CSL Tableaux

# CSL-Tableaux Proofs

Def: A CSL-Tableaux proof of A from Σ is a complete closed tableaux whose initial list comprises the members of Σ and the negation of A

Def: A CSL-proof theoretic consequence of Σ

($\Sigma \vdash_{CSL} A$) iff there is a complete closed tableaux whose initial list comprises the members of Σ and the negation of A

# Soundness and completeness

Soundness Theorem: For finite Σ, if $\Sigma|-_{CSL} A$ then $\Sigma|=_{CSL} A$

Completeness Theorem: For finite Σ, if $\Sigma|=_{CSL} A$ then $\Sigma|-_{CSL} A$

For proofs see Priest sec 1.11

Consequence of Soundness and Completeness: If we have an complete tableaux whose initial list comprises the members of Σ and the negation of A, which is open, then it is not the case that $\Sigma|-_{CSL} A$. We can therefore use tableaux to establish invalidity as well as validity.

Proof. See extra excercise

# Exercises

Using Tableaux proofs, establish which of the following are true. If an inference is CSL-invalid, read off a counter CSL-interpretation from the relevant tableaux.

(1) $\sim p \vee q, \sim r \vee q \;|\!-_{CSL} \sim(p \vee r) \vee q$

(2) $\sim p \vee (q \,\&\, r), \sim p \;|\!-_{CSL} \sim p$

(3) $|\!-_{CSL} \sim(\sim(\sim p \vee q) \vee q) \vee q)$

Extra: Priest, Ch1, Ex 5