Classical Sentential Logic and Conditionals

Seminar 3

PHIL2520 Philosophy of Logic

9 October 2012

Administration

- First assignment given out in class next week (16 October)
- Due Monday 22 October in the philosophy office by 5 PM
- Note: **No late assignments will be accepted**
- Exam date: 1.30-3.30 December 18 CPD 2.14

Reading

Required reading for this seminar: Priest, Ch 1

Required reading for next week: Priest, Ch 2

The language S

Due to the complexity of natural languages such as English, logicians study logical validity and logical consequence in simpler formal languages, such as S.

The symbols of S are:

i) Sentence constants (Priest calls them 'propositional parameters') p0, p1, p2,…

ii) '~' meaning 'it is not the case that'

iii) '&' meaning 'and'

iv) 'v' meaning 'or'

The language S (cont)

The sentences of S are:

i) This sentential constants p0, p1, p2,…

ii) The strings of symbols that can be generated by the following rule:

If A and B are formulas in S then ~A, (A & B), (A v B) are sentences in S.

A meaning interpretation m of S is a function that maps each sentence constant p to a proposition

CSL-interpretations of S

Def: A **CSL-interpretation** v of S is a function that maps each sentence constant in S to either 1 (representing truth) or 0 (representing falsehood)

If v is an CSL-interpretation of S, v is extended to all sentences of S so that it maps all sentences to either 1 or 0 in such a way that:

i) v(~A)=1 iff v(A)=0

ii) v(A&B)=1 iff v(A)=1 and v(B)=1

iii) v(A v B)=1 iff either v(A)=1 or v(B)=1

CSL-semantic consequence on S

Def: A is true under a CSL-interpretation v iff v(A)=1

Let A be a sentence in S, and Σ be a set of sentences in S.

Def: A is a **CSL-semantic consequence** of the sentences in Σ ($\Sigma \models_{CSL} A$) iff, for any CSL-interpretation v of S, if all the members of Σ are true under v then A is true under v

Logical consequence and CSL-semantic consequence

Classical Assumption: Every meaning (appropriate for a sentence) is either true or false (and not both!)

Given the classical assumption, we have:

(CSL-Equivalence) A is a CSL-semantic consequence of the sentences in Σ ($\Sigma \models_{CSL} A$) iff A is a logical consequence of the sentences in Σ

See course webpage for argument

CSL-Tableaux for S

It would be good to have a way of proving that A is a CSL-semantic consequence of Σ, or that A is not a CSL-semantic consequence of Σ.

CSL-Tableaux proofs provide a way of doing this (see Priest, sec 1.4 and 1.5).

Definitions: Tableaux, rules of generating CSL Tableaux, branches, complete CSL Tableaux, closed/open branches, closed/open CSL Tableaux

CSL-Tableaux Proofs

Def: A CSL-Tableaux proof of A from Σ is a complete closed tableaux whose initial list comprises the members of Σ and the negation of A

Def: A is a CSL-proof theoretic consequence of Σ ($\Sigma \vdash_{CSL} A$) iff there is a complete closed tableaux whose initial list comprises the members of Σ and the negation of A

Soundness and completeness

Soundness Theorem: For finite Σ, if $\Sigma \vdash_{CSL} A$ then $\Sigma \models_{CSL} A$

Completeness Theorem: For finite Σ, if $\Sigma \models_{CSL} A$ then $\Sigma \vdash_{CSL} A$

For proofs see Priest sec 1.11

Consequence of Soundness and Completeness: If we have an complete tableaux whose initial list comprises the members of Σ and the negation of A, which is open, then it is not the case that $\Sigma \vdash_{CSL} A$. We can therefore use tableaux to establish invalidity as well as validity.

Proof. See exercise 5 ch 1

Exercises

Using Tableaux proofs, establish which of the following are true. If an inference is CSL-invalid, read off a counter CSL-interpretation from the relevant tableaux.

(1) $\sim p \vee q, \sim r \vee q \; |\!-_{CSL} \sim(p \vee r) \vee q$

(2) $\sim p \vee (q \,\&\, r), \sim p \; |\!-_{CSL} \sim p$

(3) $|\!-_{CSL} \sim(\sim(\sim p \vee q) \vee q) \vee q)$

(4) Priest, Ch1, Ex 5

Conditionals

A conditional relates one proposition (the consequent) to another proposition (the antecedent) on which (in some sense) it depends.

Conditionals are expressed in English by `If'.

Examples of conditionals

- If the branch breaks the cradle will fall
- If the branch were to break, the cradle would fall
- If the cheese has been eaten, there are mice in the house
- If Susie is in New York, she is not in Hong Kong

The hook theory of conditionals

Def: ‘A ⊃ B’ is true iff ‘~A v B’ is true. (‘⊃’ is called ‘the material conditional’ or ‘hook’)

Truth table:

		A ⊃ B
A	B	T
A	~B	F
~A	B	T
~A	~B	T

The hook theory of conditionals: `If A, B’ is true iff ‘A ⊃ B’ is true

The Or-to-If argument for the hook theory of conditionals

i) Suppose `C v B' is true. Then 'If ~C, B' is true.

Example: Suppose either the gardener is the murderer or the butler is the murderer. Then, if the gardener is not the murderer, then the butler is the murderer.

Replacing C with ~A, we get: If '~A v B' is true then 'If ~~A, B' is true.

But 'If ~~A, B' is true iff 'If A, B' is true.

Hence: If '~A v B' is true then 'If A, B' is true.

The Or-to-If argument for the hook theory of conditionals (cont)

ii) Suppose ‘If A, B’ is true. Either ‘~A’ is true or ‘A’ is true. In the first case ‘~A v B’ is true. In the second case, ‘B’ is true (by modus ponens). Hence, again ‘~A v B’ is true. Hence, if ‘If A, B’ is true then ‘~A v B’ is true.

Conclusion: Putting together i) and ii) we get that ‘If A, B’ is true iff ‘A ⊃ B’ is true

Adding ⊃ and ≡

Let S* be the language obtained from S by adding ⊃ and ≡ to S, where

‘A ≡ B’ is true iff ‘(A ⊃ B) & (B ⊃ A)’ is true

Truth table:

		A ≡ B
A	B	T
A	~B	F
~A	B	F
~A	~B	T

CSL-interpretations of S*

Def: A **CSL-interpretation** v of S is a function that maps each sentence constant in S to either 1 (representing truth) or 0 (representing falsehood)

If v is an CSL-interpretation of S, v is extended to all sentences of S so that it maps all sentences to either 1 or 0 in such a way that:

i) v(~A)=1 iff v(A)=0

ii) v(A&B)=1 iff v(A)=1 and v(B)=1

iii) v(AvB)=1 iff either v(A)=1 or v(B)=1

iv) v(A⊃B)=1 iff either v(A)=0 or v(B)=1

v) v(A≡B)=1 iff v(A)=v(B)

CSL-semantic consequence on S*

Same definition as for S

CSL-Tableaux for S*

The same as for S except we also have rules for ‘$\supset$’ and ‘$\equiv$’. (See Priest Sec 1.4.)

Exercises

(5) Ex 1a Ch1, Priest

(6) Ex 1d Ch 1, Priest