Conditionals

Seminar 4

PHIL2520 Philosophy of Logic

16 October 2012

Administration

- Assignment 1: Due Monday 22 October in the philosophy office by 5 PM
- Note: **No late assignments will be accepted**
- Pick up assignments from philosophy office from wed 24 Oct
- My office hours are 2.30pm-3.30pm (or by arrangement)

Required reading: Ch1, Priest

Optional reading: Ch2-3, Grice (course website)

Reading for next seminar: Ch 2 Priest

Conditionals

A conditional relates one proposition (the consequent) to another proposition (the antecedent) on which (in some sense) it depends.

Conditionals are expressed in English by `If'.

Examples of conditionals

- If the branch breaks the cradle will fall
- If the branch were to break, the cradle would fall
- If the cheese has been eaten, there are mice in the house
- If Susie is in New York, she is not in Hong Kong

The hook theory of conditionals

Def: ‘A ⊃ B’ is true iff ‘~A v B’ is true. (‘⊃’ is called ‘the material conditional’ or ‘hook’)

Truth table:

		A ⊃ B
A	B	T
A	~B	F
~A	B	T
~A	~B	T

The hook theory of conditionals: \`If A, B’ is true iff ‘A ⊃ B’ is true

The Or-to-If argument for the hook theory of conditionals

i) Suppose `C v B’ is true. Then ‘If ~C, B’ is true.

Example: Suppose either the gardener is the murderer or the butler is the murderer. Then, if the gardener is not the murderer, then the butler is the murderer.

Replacing C with ~A, we get: If ‘~A v B’ is true then ‘If ~~A, B’ is true.

But ‘If ~~A, B’ is true iff ‘If A, B’ is true.

Hence: If ‘~A v B’ is true then ‘If A, B’ is true.

The Or-to-If argument for the hook theory of conditionals (cont)

ii) Suppose ‘If A, B’ is true. Either ‘~A’ is true or ‘A’ is true. In the first case ‘~A v B’ is true. In the second case, ‘B’ is true (by modus ponens). Hence, again ‘~A v B’ is true. Hence, if ‘If A, B’ is true then ‘~A v B’ is true.

Conclusion: Putting together i) and ii) we get that ‘If A, B’ is true iff ‘A ⊃ B’ is true

Adding ⊃ and ≡

Let S* be the language obtained from S by adding ⊃ and ≡ to S, where

‘A ≡ B’ is true iff ‘(A ⊃ B) & (B ⊃ A)’ is true

Truth table:

		A ≡ B
A	B	T
A	~B	F
~A	B	F
~A	~B	T

Meaning interpretations of S*

A meaning interpretation of S* assigns each sentence constant in S* a proposition, and each logical constant its standard meaning.

Example: Let m be a meaning interpretation of S* which assigns p_{17} to the proposition that snow is white, and p_{25} to the proposition that grass is green.

Then, under m, '$p_{17} \supset p_{25}$' expresses the proposition that not snow is white or grass is green.

Metaphysical and logical consequence in S*

Def: Q is a **metaphysical consequence** of P1,…, Pn iff, necessarily, if P1,…, Pn are all true, then Q is true

Def: Q is a **logical consequence** of P1,…,Pn iff, for any meaning interpretation m, Q is a metaphysical consequence of P1,…,Pn under m

CSL-interpretations of S*

Def: A **CSL-interpretation** v of S is a function that maps each sentence constant in S to either 1 (representing truth) or 0 (representing falsehood)

If v is an CSL-interpretation of S, v is extended to all sentences of S so that it maps all sentences to either 1 or 0 in such a way that:

i) v(~A)=1 iff v(A)=0

ii) v(A&B)=1 iff v(A)=1 and v(B)=1

iii) v(AvB)=1 iff either v(A)=1 or v(B)=1

iv) v(A⊃B)=1 iff either v(A)=0 or v(B)=1

v) v(A≡B)=1 iff v(A)=v(B)

CSL-semantic consequence on S*

Def: A is true under a CSL-interpretation v iff v(A)=1

Let A be a sentence in S*, and Σ be a set of sentences in S*.

Def: A is a **CSL-semantic consequence** of the sentences in Σ ($\Sigma \models_{CSL} A$) iff, for any CSL-interpretation v of S, if all the members of Σ are true under v then A is true under v

Logical consequence and CSL-semantic consequence

Classical Assumption: Every meaning (appropriate for a sentence) is either true or false (and not both!)

Given the classical assumption, we have:

(CSL-Equivalence for S*) For any sentence in S*, A is a CSL-semantic consequence of the sentences in Σ ($\Sigma \models_{CSL} A$) iff A is a logical consequence of the sentences in Σ

Proof. Similar to proof for S (given on course webpage)

CSL-Tableaux for S*

The same as for S except we also have rules for ‘$\supset$’ and ‘$\equiv$’. (See Priest Sec 1.4.)

Def: A CSL-tableaux for the inference from Σ to A is a complete tableaux whose initial list comprises the members of Σ and ~A.

CSL-Tableaux Proofs in S*

Def: A CSL-Tableaux proof of A from Σ is a **closed** tableaux for the inference from Σ to A

Def: A is a CSL-proof theoretic consequence of Σ ($\Sigma \vdash_{CSL} A$) iff there is a **closed** tableaux for the inference from Σ to A

Lemma: If there is a **closed** tableaux for the inference from Σ to A, then every tableaux for the inference from Σ to A is **closed**. (See Ex 5 p. 19 Priest)

Soundness and completeness in S*

Soundness Theorem: For finite Σ, if $\Sigma|\!-_{CSL} A$ then $\Sigma|\!=_{CSL} A$

Completeness Theorem: For finite Σ, if $\Sigma|\!=_{CSL} A$ then $\Sigma|\!-_{CSL} A$

For proofs see Priest sec 1.11

Objection 1 to the hook theory of conditionals

According to the hook theory:

‘If A, B’ is true iff ‘~A v B’ is true

Hence, according to the hook theory, (1-3) are true.

(1) If Hong Kong is in New Zealand then grass is green

(2) If Hong Kong is a city then World War II ended in 1945

(3) If World War II ended in 1941 then 2+2=5

But (1-3) all seem false.

Towards Grice's response: Rules of Conversation

Conversation has rules, which need to be obeyed if the aims of conversation are going to be met.

An important rule (Strength): Assert the strongest relevant claim that you are in a position to make

If this rule is broken, then participants will often be misled.

Example

Suppose Jane knows that John is in the pub.

Clare to Jane: Where is John?

Jane to Clare: He is either in the pub or in the library

Jane's assertion is true, but misleading.

It is misleading since it will lead Clare to falsely believe that Jane doesn't know whether John is in the pub

Grice’s defence of the hook theory

Asserting (1) is inappropriate since asserting it would break the rule of strength, since we know that grass is green.

Similarly, (2-3) are also inappropriate to assert.

Grice: We mistakenly think (1-3) to be false since we mistake inappropriateness with falsity.

Objection 2: Indicative vs Subjunctive conditionals

There are pairs of conditionals that appear to have the same antecedent and consequent, but differ in truth value.

(4) If Oswald didn't shoot Kennedy someone else did (True) (Indicative conditional)

(5) If Oswald hadn't shot Kennedy someone else would have (False) (Subjunctive conditional)

Therefore: The hook theory cannot work for all conditionals

Response to objection 2

The hook theory works for indicative conditionals, but not for subjunctive conditionals.

Objection 3: More counterexamples

$(A\&B)\supset C \models_{CSL} (A\supset C)v(B \supset C)$

Hence, according to the hook theory, (6) is a logically valid argument. But it appears invalid.

(6) If you close switch x and switch y, the light goes on. Hence, it is the case either that if you close the switch x the light will go on, or that if you close switch y the light will go on.

Objection 3: More counterexamples (cont)

Conversational rules don’t seem to be able to explain why this argument seems bad

Example: Uttering (6) does not break the rule of strength

See Priest p 15 for further problematic arguments for the hook theory