Normal Modal Logic 2

Seminar 8

PHIL2520 Philosophy of Logic

27 November 2012

Administration

- Assignment 2 handback session: 3.30-4.30pm Philosophy seminar room (Attendance optional)
- Marked assignments can be picked up from the philosophy office any time afterwards
- Solutions to Assignment 2 will be put on the course website
- Assignment 3: Due Monday 3 December 5pm Philosophy Office
- Required reading for this seminar: Ch3, Priest

Schedule for rest of course

4 Dec: Conditionals 2 (reading: Priest Ch4 sec 4.5-4.9; Ch5)

11 Dec: Many Valued Logic (reading: Priest Ch 7)

18 Dec: Exam 1.30pm-3.30pm here

8 Jan: Essay Due 5pm Email submission to danm@hku.hk (Make sure you write your name and student number in the email title and on your essay and format with double spaces and standard 1 inch size margins)

The language M

M is the language obtained from S* by adding '□' and '◊', where

i) '□' symbolises 'necessarily',

ii) '◊' symbolises 'possibly', and

iii) '□A' and '◊A' are sentences in M if A is a sentence in M.

K models

A K model of M is a triple <W, R,v>, where

i) W is a non-empty set (the set of “possible worlds”),

ii) R is a binary set-relation on W (the “relative possibility relation”), and

iii) v is a function mapping each pair comprising of a sentential constant p and world w to either 0 or 1 (denoted by $v_w(p)$).

v is called the interpretation wrt <W, R, v>, while R is called the **accessibility relation** wrt <W, R, v>.

K models (cont)

Given an interpretation v, v is extended to apply to all sentences in M so that it maps all sentences to either 1 or 0 in such a way that:

i) $v_w(\sim A)=1$ iff $v_w(A)=0$

ii) $v_w(A\&B)=1$ iff $v_w(A)=1$ and $v_w(B)=1$

iii) $v_w(AvB)=1$ iff either $v_w(A)=1$ or $v_w(B)=1$

iv) $v_w(A\supset B)=1$ iff either $v_w(A)=0$ or $v_w(B)=1$

v) $v_w(A\equiv B)=1$ iff $v_w(A)=v_w(B)$

vi) $v_w(\Box A)=1$ iff, for any w' in W such that w stands in R to w', $v_{w'}(A)=1$, and

vii) $v_w(\Diamond A)=1$ iff, for some w' in W such that w stands in R to w', $v_{w'}(A)=1$

Kc models

Let c be a constraint on accessibility relations R. Then a Kc model is a K model <W, R, v> where R satisfies the constraint c.

Def: Q is a **Kc-semantic consequence** of P1,…,Pn (P1,…,Pn $|=_{Kc}$ Q) iff, for any K-model <W, R, v>, for any w in W, if P1,…, Pn are true at w under <W, R, v>, then Q is true at w under <W, R, v>.

Possible constraints

ρ (rho), reflexivity: for any w ∈ W, <w,w> ∈ R

σ (sigma), symmetry: for any w,w’ ∈ W, if <w,w’> ∈ W, then <w’,w> ∈ R

τ (tau), transitivity: for any w,w’,w’’ ∈ W, if <w,w’> ∈ W and <w’,w’’> , then <w,w’’> ∈ R

η (eta), extendability: for any w ∈ W, there is a w’ ∈ W such that <w,w’> ∈ R

Logical soundness and logical completeness

Def: Logic X is **logically sound** for language M iff: For any sentences P1,…,Pn, Q in M, if Q is a K-semantic consequence of P1,…,Pn, then Q is a logical consequence of P1,…,Pn

Def: Logic X is **logically complete** for language M iff: For any sentences P1,…,Pn, Q in M, if Q is a logical consequence of P1,…,Pn, then Q is a a K-semantic consequence of P1,…,Pn

What is the true logic for M?

Def: Logic X is the true logic for M iff X is logically sound and complete for language M

- In order to find the true logic for M we need to determine what constraints accessibility relations R should satisfy
- To answer this question we need to determine what constraints the relation of relative possibility satisfies

A popular answer

A popular answer is that the relation of relative possibility is reflexive, symmetric and transitive (that is, it satisfies ρ, σ, and τ)

Given this answer, and given (necessary bivalence), we can establish (Result 2).

(Result 2) For any sentences P1,…,Pn, Q in M,

Q is a logical consequence of P1,…,Pn iff Q is a a Kρστ-semantic consequence of P1,…,Pn

It follows from (Result 2) that Kρστ is the true logic for M.

The specialness of logic S5

Consider the following trivial constraint:

ν (nu) triviality: for any w,w'∈W, <w,w'> ∈ W

In a ν-model, R becomes redundant: we could replace ν-models with <W,v> pairs and drop all mention of R.

Result: Kρστ is equivalent to Kν in the sense that P1,…,Pn $|=_{K\rho\sigma\tau}$ Q iff P1,…,Pn $|=_{K\nu}$ Q (See Priest for proof.)

Because of this equivalence Kρστ and Kν are both called S5 and, given (Result 2) the true logic of M is S5

Reason 1 to reject S5: rejecting symmetry

Combinatorialism: p is possible iff it results from recombining actual things and actually instantiated properties (Armstrong, Wittgenstein)

Ex: ‘There is a blue sheep’ is possible since the property of being a sheep can be combined with the property of being blue

Given combinatorialism, it is plausible that there can’t be more things than there actually are, but there can be less things. Given this, symmetry fails

Reason 2 to reject S5: Reject transitivity

Chisholm's Paradox:

(1) I can be transformed into a poached egg by small changes

(2) If only a small change would make me an F, then I could have been an F

(3) Relative possiblity is transitive

It follows from (1-3) that I could have been a poached egg. But this is false – I could not have been a poached egg!

Solution endorsed by David Lewis

The relative possiblity relation is not transitive.

Why isn't relative possiblity transitive?

Lewis's answer: possiblity is a matter of similarity (I am possibly F iff being F is not too different from how I actually am). But similarity is not transitive.

Tableaux rules for ρ, σ, τ, η

See p38-45 Priest

Soundness and completeness of Kc Tableaux proofs wrt Kc

Suppose c is either ρ, σ, τ, η, or any combination of them.

Theorem: Kc tableaux proofs are **sound** wrt Kc (If Q is a Kc proof theoretic of P1,…,Pn then Q is a Kc semantic consequence of P1,…Pn)

Theorem: Kc tableaux proofs are **complete** wrt Kc (If Q is a Kc semantic theoretic of P1,…,Pn then Q is a Kc proof theoretic consequence of P1,…Pn)

See Priest for proof.

Nomic language N

Language N is the same as M except ‘□’ in N symbolises ‘It is physically necessary that’, and ‘◊’ symbolises ‘It is physically possible that’, where:

i) It is physically necessary that A iff it follows from the physical laws that A;

ii) It is physically possible that A iff it is consistent with the physical laws that A

A possible worlds analysis of physical necessity and physical possibility

i) ‘It is physically **necessary** that A’ is true at possible world w iff, for **any** possible world w’ that is compatible with the physical laws at w, A is true at w’;

ii) ‘It is physically **possible** that A’ is true at possible world w iff, for **some** possible world w’ that is compatible with the physical laws at w, A is true at w’;

What is the true logic of language N?

Def: Q is a **logical consequence** of P1,…,Pn in N iff, for any meaning interpretation m, for any possible world w, if P1,…, Pn are true at w under m, then Q is true at w under m

Hypothesis: The true logic for N is a normal modal logic

Question: If this hypothesis is correct, which normal modal logic is the true logic for N?

The temporal language T

Language T is the same as M except '□' in T symbolises 'It will always be the case that', and '◊' symbolises 'It will be the case that'.

The possible worlds analysis of tense:

i) 'It will always be the case that A' is true at world t iff, for **any** world t' that is future relative to t, A is true at t'

ii) 'It will be the case that A' is true at world t iff, for **some** world t' that is future relative to t, A is true at t'

where t' is future relative to t iff, according to t, t' will obtain

The epistemic language Ex

Language Ex is the same as M except ‘□’ in T symbolises ‘x knows that’, and ‘◊’ symbolises ‘x does not know that it is not the case that’.

The possible worlds analysis of knowledge:

i) ‘x knows that A’ is true at world t iff, for **any** world t’ that is compatible with x’s knowledge at t, A is true at t’

The obligation language O

Language O is the same as M except '□' in T symbolises 'It obligatory that', and '◊' symbolises 'It is permissible that'.

The possible worlds analysis of obligation:

i) 'It is obligatory that A' is true at world t iff, for **any** world t' at which nothing impermissible happens, A is true at t'

ii) 'It is permissible that A' is true at world t iff, for **some** world t' at which nothing impermissible happens, A is true at t'

What is the true logic of languages T, E and O?

Hypothesis: The true logics for T, E and O are normal modal logics

Question: If this hypothesis is correct, which normal modal logics are the true logics for T, E and O?