

Solutions to Assignment 2

21. Show that, for any K-model $\langle W, R, v \rangle$, for any $w \in W$, and for any sentence A in \mathcal{M} :

$$v_w(\neg \Box A) = v_w(\Box \neg A)$$

Proof. ~~Let~~ Let $\langle W, R, v \rangle$ be a K-model, $w \in W$, and A be a sentence in \mathcal{M} . Then:

$$v_w(\neg \Box A) = 1 \quad \text{iff}$$

there is a $w' \in W$ such that $\underbrace{w, w' \in R \text{ and}}_{\text{and}} v_{w'}(A) = 0$

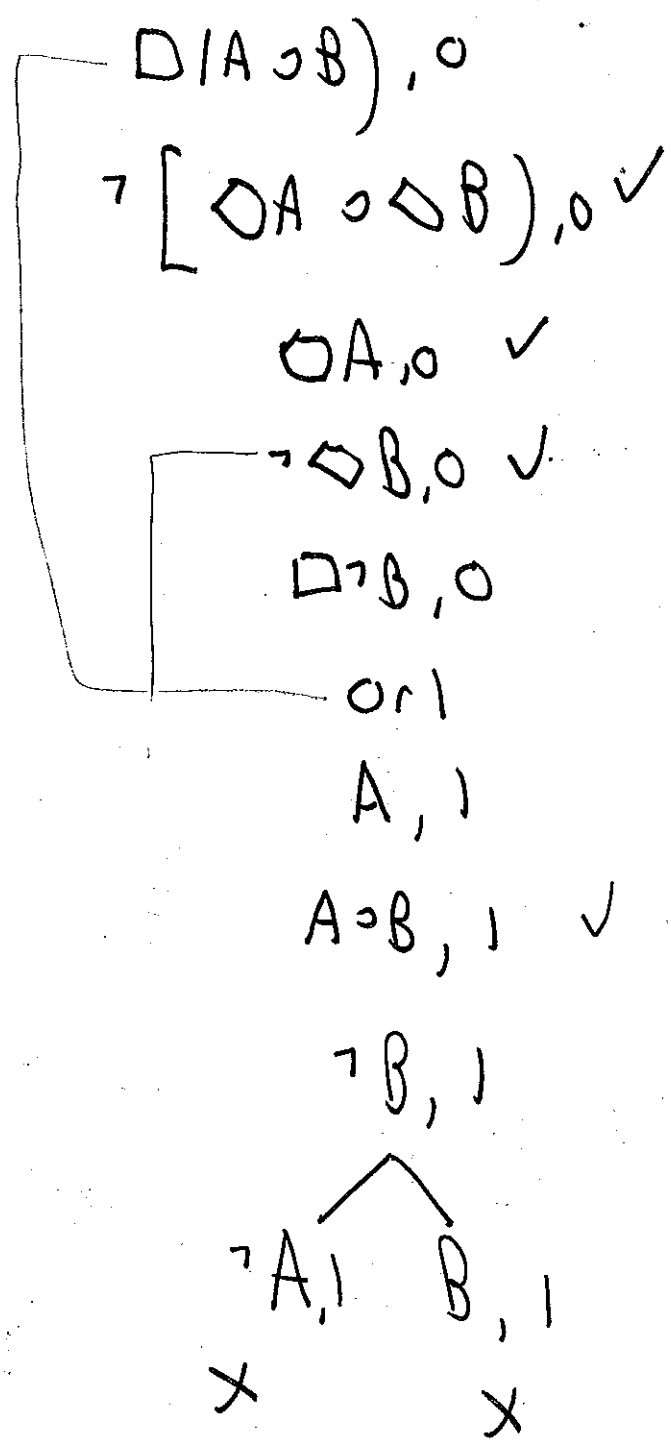
iff there is a $w' \in W$ such that $\underbrace{w, w' \in R \text{ and}}_{\text{and}} v_{w'}(\neg A) = 1$

$$\text{iff } v_w(\Box \neg A) = 1$$

Q2. ~~2)~~ Sh Establish

$$\Box(A \supset B) \vdash \Box A \supset \Box B$$

Proof.



Q3

Establich $\vdash DA \equiv D(\neg A \supset A)$

Proof.

$\checkmark \neg [DA \equiv D(\neg A \supset A)], 0$

$\square A, 0$
 $\neg D(\neg A \supset A), 0$
 $\square D(\neg A \supset A), 0$
 $0, 1$

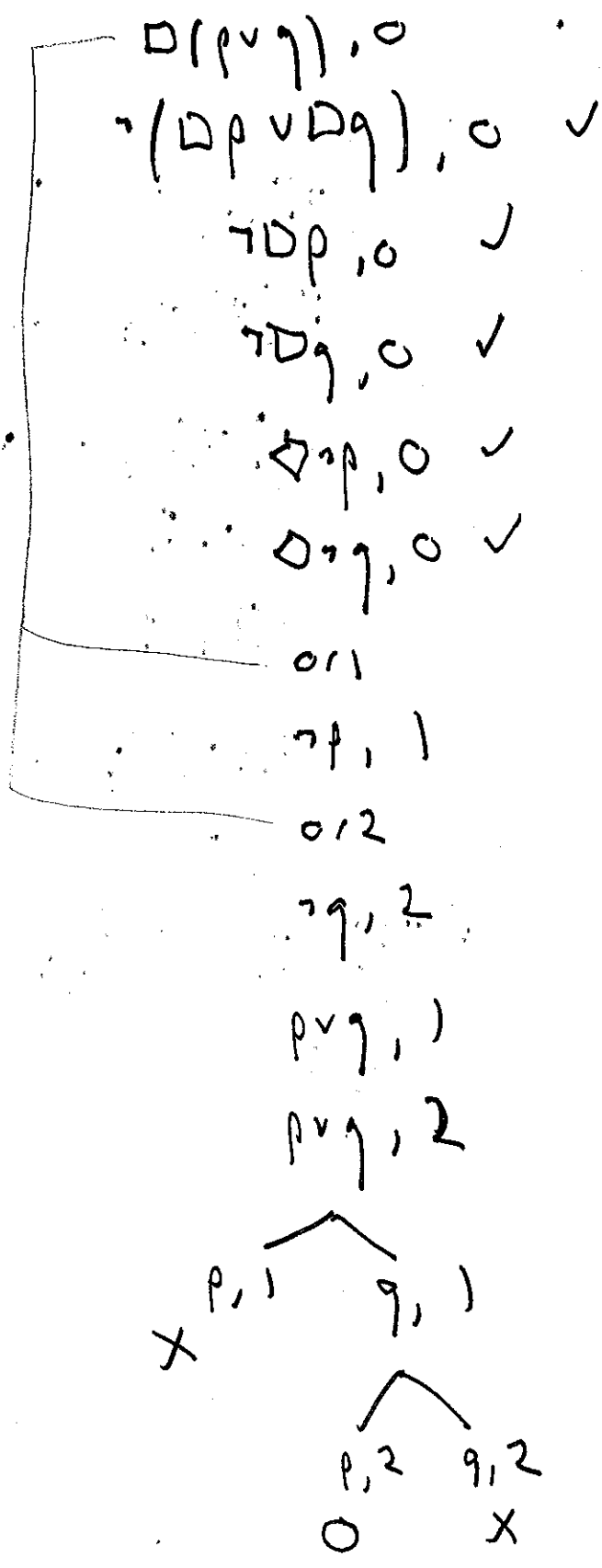
$\checkmark \neg(\neg A \supset A), 1$
 $\neg A, 1$
 $\neg A, 1$
 $A, 1$
 \times

$\neg DA, 0 \checkmark$
 $D(\neg A \supset A), 0$
 $\square \neg A, 0 \checkmark$
 $0, 2$
 $\neg A, 2$
 $\neg A \supset A, 2 \checkmark$
 $\checkmark \neg \neg A, 2$ $A, 2$
 $\neg A, 2$ \times
 \times

Q4 Establish and find and verify a counter model.

~~Q4~~ $\neg (D(p \vee q) \supset (Dp \vee Dq))$

Proof. $\neg [D(p \vee q) \supset (Dp \vee Dq)]$, 0 ✓



Countermodel:

$$\text{Let } W = \{w_0, w_1, w_2\}$$

$$R = \{ \langle w_0, w_1 \rangle, \langle w_0, w_2 \rangle \}$$

$$v_{w_2}(p) = 1 \quad v_{w_2}(q) = 0, \quad v_{w_1}(p) = 0, \quad v_{w_1}(q) = 1$$

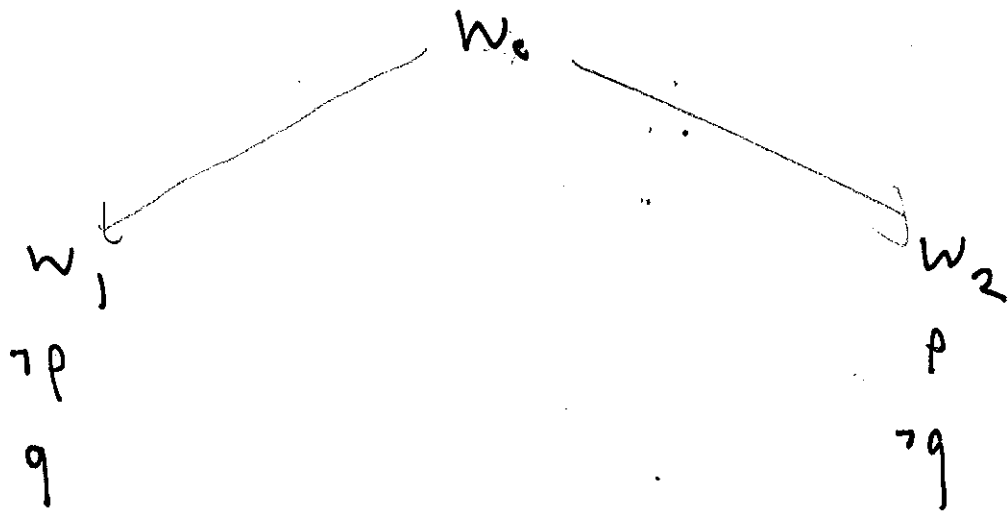
$$v_{w_0}(p) = 1$$

Verify this is a countermodel

We have

$$\neg Dq \quad \neg (Dp \vee Dq)$$

$$D(p \vee q)$$



Hence ' $D(p \vee q) \supset (Dp \vee Dq)$ ' is false

5. Establish and find and verify a countermodel

~~2.5~~ ~~$\forall p \supset \exists p$~~ $\forall p \supset \exists p$

Proof.

$$\neg [\forall p \supset \exists p], 0 \quad \checkmark$$

- $\forall p, 0$
- $\exists p \forall p, 0 \quad \checkmark$
- $\forall p \exists p, 0 \quad \checkmark$
- or
- $\exists p \forall p, 1 \quad \checkmark$
- $\forall p \exists p, 1$
- 1, 2
- $\exists p, 2$
- $p, 1$
- 0

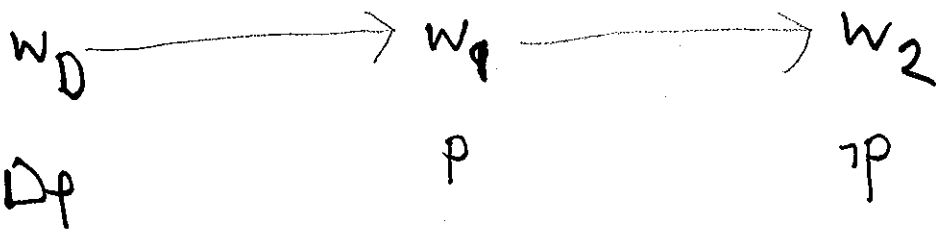
Hence $\times (\forall p \supset \exists p)$

Continued)

$$\text{Let } W = \{w_1, w_2, w_3\}$$

$$R = \{ \langle w_0, w_1 \rangle, \langle w_1, w_2 \rangle \}$$

$$v_{w_1}(p) = 1, \quad v_{w_2}(p) = 0$$



\Rightarrow DDP

Hence $(Dp \Rightarrow DDP)$ is ~~the~~ false at w_0 under $\langle W, R, v \rangle$