Assignment 3

1. Ex 1(a) Ch3.

i) R_1 satisfies ρ, but not σ, τ)

$R_1 = \{ \langle w_1, w_1 \rangle, \langle w_2, w_2 \rangle, \langle w_3, w_3 \rangle, \langle w_1, w_2 \rangle, \langle w_2, w_3 \rangle \}$

$W = \{w_1, w_2, w_3\}$

ii) R_2 satisfies σ, but not η, ρ, or τ

$R_2 = \{ \langle w_1, w_2 \rangle, \langle w_2, w_1 \rangle, \langle w_2, w_3 \rangle, \langle w_3, w_2 \rangle \}$

$W = \{w_1, w_2, w_3, w_4\}$

iii) R_3 satisfies τ, but not ρ, η, or σ

$R_3 = \{ \langle w_1, w_2 \rangle \}$

$W = \{w_1, w_2\}$

iv) R_ψ satisfies η, but ρ, σ or τ

$R_\psi = \{ \langle w_1, w_2 \rangle, \langle w_2, w_3 \rangle, \langle w_3, w_1 \rangle$

$W = \{ w_1, w_2, w_3 \}$

2. Ex 4a ch3 ~~the find countermodel and verify it is a countermodel)~~

Establish $\vdash_{K\rho\tau} (\Box A \vee \Box B) \equiv \Box(\Box A \vee \Box B)$

$\neg[(\Box A \vee \Box B) \equiv \Box(\Box A \vee \Box B)], 0$ ✓

✓ $\Box A \vee \Box B, 0$

✓ $\neg\Box(\Box A \vee \Box B), 0$

✓ $\Diamond\neg(\Box A \vee \Box B), 0$

$0r1$

✓ $\neg(\Box A \vee \Box B), 1$

$0r0, 1r1$

✓ $\neg\Box A, 1$

✓ $\neg\Box B, 1$

$\Diamond\neg A, 1$

$\Diamond\neg B, 1$

$1r2$

$\neg A, 2$

$1r3$

$\neg B, 3$

$2r2, 3r3, 0r2, 0r3$

$\Box A, 0$ | $\Box B, 0$

$A, 2$ | $B, 3$

× | ×

$\neg(\Box A \vee \Box B), 0$ ✓

$\Box(\Box A \vee \Box B), 0$

$\neg\Box A, 0$ ✓

$\neg\Box B, 0$ ✓

$\Diamond\neg A, 0$ ✓

$\Diamond\neg B, 0$ ✓

$0r1$

$\neg A, 1$

$0r2$

$\neg B, 2$

$0r0, 1r1, 2r2$

* $\Box A \vee \Box B, 0$ ✓

$\Box A, 0$ | $\Box B, 0$

$A, 1$ | $B, 2$

× | ×

3. ES a) Ch3

Establish $\vdash_{K\rho}(\Diamond A \supset \Diamond\Diamond A)$

$\neg(\Diamond A \supset \Diamond\Diamond A), 0$ ✓

$\Diamond A, 0$ ✓

$\neg\Diamond\Diamond A, 0$ ✓

$A, 1$

$\Box\neg\Diamond A, 0$

$\neg\Diamond A, 1$ ✓

$\Box\neg A, 1$

$\neg A, 1$

×

4. Establish whether (*) is true. If it is false, find a countermodel and verify it is a countermodel)

(*) $\vdash_{K\rho\tau}(\Diamond\Box p \supset \Box\Diamond p)$

$\neg(\Diamond\Box p \supset \Box\Diamond p), 0$ ✓

$\Diamond\Box p, 0$ ✓

$\neg\Box\Diamond p, 0$ ✓

$\Diamond\neg\Diamond p, 0$

$0r1$

$\Box p, 1$

$0r0, 1r1, 0r2$

$\neg\Diamond p, 2$ ✓

$2r2,$

$\Box\neg p, 2$

$p, 1$

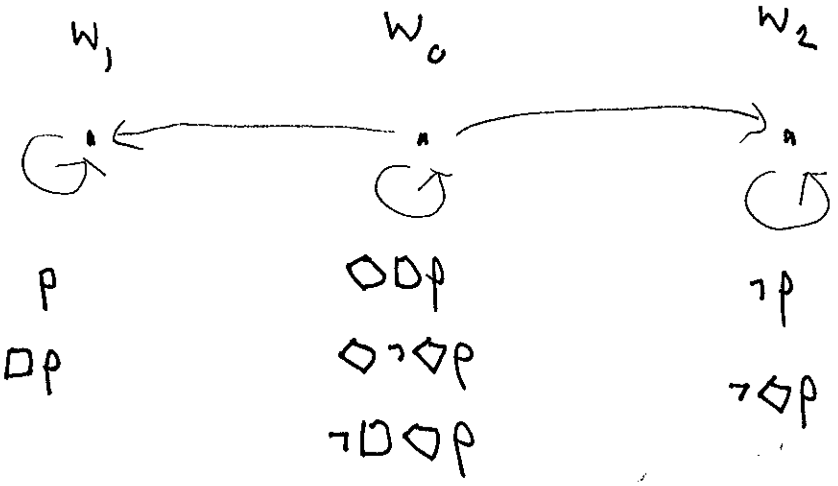
$\neg p, 2$

○

Hence (*) is false.

4. (continued)

The countermodel is:



Hence $\ulcorner \Diamond\Box p \supset \Box\Diamond p \urcorner$ is false at w_0 in $\langle W, R, v \rangle$

where

$W = \{w_0, w_1, w_2\}$

$R = \{\langle w_1, w_1\rangle, \langle w_2, w_2\rangle, \langle w_0, w_0\rangle, \langle w_0, w_1\rangle, \langle w_0, w_2\rangle\}$

$v_{w_1}(p) = 1$, $v_{w_2}(p) = 0$

5. Ex 7b) Ch3

i) Proof that $K\rho$ is not an extension of $K\sigma$ or $K\tau$

~~$\vdash_{K\rho} \Box p \supset p$~~ $\vDash_{K\rho} (\Box p \supset p)$

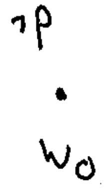
but $\nvDash_{K\sigma} (\Box p \supset p)$ and $\nvDash_{K\tau} (\Box p \supset p)$

Countermodel of '$\Box p \supset p$' in $K\sigma$ and $K\tau$:

~~$W = \{w_1, w_2\}$~~

~~$R = \{\langle w_1, w_2\rangle, \langle w_2, w_1\rangle\}$~~

~~$V_{w_1}(p) = 1, V_{w_2}(p) = 0$~~



$W = \{w_0\}$

$R = \emptyset$ (the empty set)

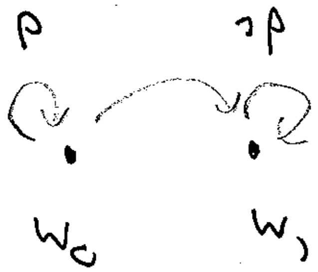
$V_{w_0}(p) = 0$

ii) Proof that $K\sigma$ is not an extension of $K\rho$ or $K\tau$

$\models_{K\sigma} (p \supset \Box\Diamond p)$, but

$\not\models_{K\rho} (p \supset \Box\Diamond p)$ and $\models_{K\tau} (p \supset \Box\Diamond p)$

Countermodels in $K\rho$ and $K\tau$:



iii) Proof that $K\tau$ is not an extension of $K\rho$ and ~~Kτ~~ $K\sigma$

$\models_{K\tau} (\Box p \supset \Box\Box p)$

but $\not\models_{K\sigma} (\Box p \supset \Box\Box p)$ and $\not\models_{K\rho} (\Box p \supset \Box\Box p)$

Countermodels in $K\sigma$ and $K\rho$