Problem Set 2

Elementary Logic

Due: 5 December 2005

Name _____

Student ID Number _____

email _____

Mark _____%

Due **5 December 2005** by **4:00PM**.

Submit your problem set to Ms. Loletta Li in Main Building 302. (If she is not available, go to room 312, the Philosophy department General Office.) Make sure your problem set is timestamped. Do not submit assignments by email. Late penalty: 10% for each day late. This problem set will not be accepted after 9 December.

Answer the questions on the problem set itself. Write neatly. If the grader cannot read your handwriting, you will not receive credit.

Be sure that all pages of the assignment are securely stapled together.

Check the course bulletin board for announcements about the assignment.

Do your own work. If you copy your problem set, or permit others to copy, you may fail the course.

1. (20 marks)
True or false?
Circle 'T' if the statement is true.
Circle 'F' if the statement is false.
For this question, you may assume that φ and ψ are SL WFFs.

T F If φ is contingent, then φ may be inconsistent.

T F If φ and ψ are logically equivalent, then ψ entails φ.

T F For any SL WFF φ, there is another SL WFF ψ, where ψ is different from φ, and ψ entails φ.

T F If φ is an inconsistent conjunction, then one of φ's conjuncts may be a tautology.

T F If X is an inconsistent set of SL wffs, then each member of X is either inconsistent or contingent.

T F 'A' is a logical consequence of '$(A \vee B)$' and '$\sim B$'.

T F Every inconsistent SL WFF contains the connective '$\sim$'.

T F If X is a consistent set of SL WFFs, then every subset of X is consistent.

T F If φ is inconsistent, then φ entails itself.

T F At least one MPL WFF is an SL WFF.

/20

2. (20 marks)
For each of the following:
Circle "tautology" if it is a WFF of SL that is a tautology.
Circle "contingent" if it is a contingent WFF of SL.
Circle "inconsistent" if it is an inconsistent WFF of SL.
Otherwise, don't circle anything.

$$((D \rightarrow A) \vee (B\& \sim C))$$
tautology contingent inconsistent

$$A \vee A$$
tautology contingent inconsistent

$$((A\&(B \vee C)) \leftrightarrow ((A \vee B)\&(A \vee C)))$$
tautology contingent inconsistent

$$((A\&B) \rightarrow ((A \vee C)\&(B \vee A)))$$
tautology contingent inconsistent

$$((A \rightarrow \sim A) \vee (\sim B\&A))$$
tautology contingent inconsistent

$$((A\& \sim A) \rightarrow (B\& \sim C))$$
tautology contingent inconsistent

$$((A \rightarrow B) \vee (B \rightarrow A))$$
tautology contingent inconsistent

$$((((C \rightarrow B) \rightarrow C) \rightarrow C) \rightarrow C)$$
tautology contingent inconsistent

$$(((C \rightarrow B) \rightarrow C) \rightarrow C)$$
tautology contingent inconsistent

$$(\sim(A \vee B) \leftrightarrow (\sim A\& \sim B))$$
tautology contingent inconsistent

/20

3. (10 marks)

Use the full truth table method to test the validity of the following two SL sequents:

(a) $((A \rightarrow (C\&D))\&(\sim A \rightarrow B)), \sim D \models B$

(b) $(B \leftrightarrow (A\&C)), (D\&(A \rightarrow \sim C)) \models \sim B$

/10

4. (10 marks)

(a) Suppose that '#' is a new two-place connective added to SL.

You are given the following information about '#':

"$(\sim A \rightarrow (A\#B))$" is a tautology.

"$(A\#B), A \models \sim B$" is a valid sequent.

"$(A\#B)$" does not entail "$\sim A$".

Write down every possible truth table which '#' might have.

(b) Write down a WFF of SL which contains only the connectives '$\sim$' and '$\vee$' and the sentence letters 'A' and 'C', and which is logically equivalent to the following:

$(((B \vee A) \vee (\sim B \vee A)) \leftrightarrow (C \& A))$

/10

5. (10 marks)

Translate the following statements into SL, preserving as much structure as possible. Be sure to write down your translation scheme.

(a) Either Lee or Sue saw us.

(b) Lee will meet you, unless you are late.

(c) If I win the lottery I will go to Madagascar, but otherwise I'll see you at the races on Saturday.

(d) Provided that Sue is elected, I won't see you and I won't write.

(e) If you think I am crazy, then you should go there only if you are not afraid.

/10

6. (10 marks)

Determine whether or not the following argument is valid by first, translating the argument into an SL sequent, and then determining whether or not the resulting sequent is valid. Be sure to write down your translation scheme.

If Lee is in Barcelona, neither Sue nor Marian is. If Marian is not in Barcelona but Sue is, then Lee is in Barcelona. Sue is in Barcelona if and only if Marian is not in Barcelona. Therefore, Marian is in Barcelona, but Lee and Sue are not.

/10

7. (10 marks)

Translate the following into MPL, preserving as much structure as possible. Be sure to write down your translation scheme.

(a) John ate the apple if someone did.

(b) If Marge and Harold both ran the marathon then Marge won.

(c) All elephants are mammals, but if Herman is an elephant then he is no mammal.

(d) Some engineers are clever. Leo is not an engineer. So Leo is not clever.

(e) If everyone is here then someone who is here will not be here tomorrow.

/10

8. (10 marks)

Which is the following is a WFF of MPL?
Circle each one that is a WFF of MPL.
For each one that is not a WFF of MPL, explain why it is not a WFF.

$(\forall x Qx \& \sim \sim \sim \forall x Qy)$

$(\exists x(Fx \rightarrow Gx) \vee \forall y \forall x(Hy \rightarrow Ha))$

$\sim(\exists x(Fx \vee Fb) \& Fa)$

$(Fa \vee \forall x(Fx \vee Fb)$

$\forall y(\exists x Fb \rightarrow Hy)$

/10