Problem Set 2

Elementary Logic

Due: 5 December 2005

Name ____________________Model Student ____________________

Student ID Number ____

email ____

Mark ___100 %

Due **5 December 2005** by **4:00PM**.

Submit your problem set to Ms. Loletta Li in Main Building 302. (If she is not available, go to room 312, the Philosophy department General Office.) Make sure your problem set is timestamped. Do not submit assignments by email. Late penalty: 10% for each day late. This problem set will not be accepted after 9 December.

Answer the questions on the problem set itself. Write neatly. If the grader cannot read your handwriting, you will not receive credit.

Be sure that all pages of the assignment are securely stapled together.

Check the course bulletin board for announcements about the assignment.

Do your own work. If you copy your problem set, or permit others to copy, you may fail the course.

1. (20 marks)
True or false?
Circle 'T' if the statement is true.
Circle 'F' if the statement is false.
For this question, you may assume that φ and ψ are SL WFFs.

T **F** If φ is contingent, then φ may be inconsistent.

T F If φ and ψ are logically equivalent, then ψ entails φ.

T F For any SL WFF φ, there is another SL WFF ψ, where ψ is different from φ, and ψ entails φ.

T F If φ is an inconsistent conjunction, then one of φ's conjuncts may be a tautology.

T **F** If X is an inconsistent set of SL wffs, then each member of X is either inconsistent or contingent.

T F 'A' is a logical consequence of '$(A \vee B)$' and '$\sim B$'.

T F Every inconsistent SL WFF contains the connective '$\sim$'.

T F If X is a consistent set of SL WFFs, then every subset of X is consistent.

T F If φ is inconsistent, then φ entails itself.

T **F** At least one MPL WFF is an SL WFF.

20/20

2. (20 marks)
For each of the following:
Circle "tautology" if it is a WFF of SL that is a tautology.
Circle "contingent" if it is a contingent WFF of SL.
Circle "inconsistent" if it is an inconsistent WFF of SL.
Otherwise, don't circle anything.

$$((D \rightarrow A) \vee (B \& \sim C))$$

tautology **contingent** inconsistent

$$A \vee A$$

tautology contingent inconsistent

$$((A \& (B \vee C)) \leftrightarrow ((A \vee B) \& (A \vee C)))$$

tautology **contingent** inconsistent

$$((A \& B) \rightarrow ((A \vee C) \& (B \vee A)))$$

tautology contingent inconsistent

$$((A \rightarrow \sim A) \vee (\sim B \& A))$$

tautology **contingent** inconsistent

$$((A \& \sim A) \rightarrow (B \& \sim C))$$

tautology contingent inconsistent

$$((A \rightarrow B) \vee (B \rightarrow A))$$

tautology contingent inconsistent

$$((((C \rightarrow B) \rightarrow C) \rightarrow C) \rightarrow C)$$

tautology **contingent** inconsistent

$$(((C \rightarrow B) \rightarrow C) \rightarrow C)$$

tautology contingent inconsistent

$$(\sim (A \vee B) \leftrightarrow (\sim A \& \sim B))$$

tautology contingent inconsistent

20/20

3. (10 marks)

Use the full truth table method to test the validity of the following two SL sequents:

(a) $((A \rightarrow (C\&D))\&(\sim A \rightarrow B)), \sim D \models B$

(b) $(B \leftrightarrow (A\&C)), (D\&(A \rightarrow \sim C)) \models \sim B$

Both (a) and (b) are valid sequents, as truth tables show

10/10

4. (10 marks)

(a) Suppose that '#' is a new two-place connective added to SL.

You are given the following information about '#':

"$(\sim A \rightarrow (A\#B))$" is a tautology.

"$(A\#B), A \models \sim B$" is a valid sequent.

"$(A\#B)$" does not entail "$\sim A$".

Write down every possible truth table which '#' might have.

A	**B**	**(A # B)**
T	**T**	**F**
T	**F**	**T**
F	**T**	**T**
F	**F**	**T**

(b) Write down a WFF of SL which contains only the connectives '$\sim$' and '$\vee$' and the sentence letters 'A' and 'C', and which is logically equivalent to the following:

$(((B \vee A) \vee (\sim B \vee A)) \leftrightarrow (C\&A))$

$\sim(\sim\mathbf{C} \vee \sim\mathbf{A})$ (other answers are possible)

10/10

5. (10 marks)

Translate the following statements into SL, preserving as much structure as possible. Be sure to write down your translation scheme.

(a) Either Lee or Sue saw us.

L: Lee saw us
S: Sue saw us
$(L \vee S)$ **(but** $((L \vee S) \& \sim (L \& S))$ **is acceptable)**

(b) Lee will meet you, unless you are late.

M: Lee will meet you
L: You are late
$(L \leftrightarrow \sim M)$ **(but** $(M \vee L)$ **is acceptable)**

(c) If I win the lottery I will go to Madagascar, but otherwise I'll see you at the races on Saturday.

W: I win the lottery
M: I will go to Madagascar
S: I'll see you at the races on Saturday
$((W \rightarrow M) \& (\sim W \rightarrow S))$

(d) Provided that Sue is elected, I won't see you and I won't write.

S: Sue is elected
C: I will see you
W: I will write
$(S \rightarrow (\sim C \& \sim W))$

(e) If you think I am crazy, then you should go there only if you are not afraid.

C: You think I am crazy
G: You should go there
A: You are afraid
$(C \rightarrow (G \rightarrow \sim A))$

10/10

6. (10 marks)

Determine whether or not the following argument is valid by first, translating the argument into an SL sequent, and then determining whether or not the resulting sequent is valid. Be sure to write down your translation scheme.

If Lee is in Barcelona, neither Sue nor Marian is. If Marian is not in Barcelona but Sue is, then Lee is in Barcelona. Sue is in Barcelona if and only if Marian is not in Barcelona. Therefore, Marian is in Barcelona, but Lee and Sue are not.

L: Lee is in Barcelona
S: Sue is in Barcelona
M: Marian is in Barcelona

$$(L \to ({\sim}S \& \sim M)), (({\sim}M \& S) \to L), (({\sim}M \& S) \to L) \models (M \& ({\sim}L \& \sim S))$$

which is a valid sequent, as can be shown by a truth table:

L	S	M	$(L \to ({\sim}S \& \sim M))$	$(({\sim}M \& S) \to L)$	$(({\sim}M \& S) \to L)$	$(M \& ({\sim}L \& \sim S))$
T	**T**	**T**	**F**	**T**	**F**	**F**
T	**T**	**F**	**F**	**T**	**T**	**F**
T	**F**	**T**	**F**	**T**	**T**	**F**
T	**F**	**F**	**T**	**T**	**F**	**F**
F	**T**	**T**	**T**	**T**	**F**	**F**
F	**T**	**F**	**T**	**F**	**T**	**F**
F	**F**	**T**	**T**	**T**	**T**	**T**
F	**F**	**F**	**T**	**T**	**F**	**F**

10/10

7. (10 marks)

Translate the following into MPL, preserving as much structure as possible. Be sure to write down your translation scheme.

(a) John ate the apple if someone did.

j: John
Ax: x ate the apple
Px: x is a person
$(\exists x(Px \& Ax) \rightarrow Aj)$

(b) If Marge and Harold both ran the marathon then Marge won.

m: Marge
h: Harold
Rx: x ran the marathon
Wx: x won
$((Rm \& Rh) \rightarrow Wm)$

(c) All elephants are mammals, but if Herman is an elephant then he is no mammal.

h: Herman
Ex: x is an elephant
Mx: x is a mammal
$(\forall x(Ex \rightarrow Mx) \& (Eh \rightarrow \sim Mh))$

(d) Some engineers are clever. Leo is not an engineer. So Leo is not clever.

o: Leo
Ex: x is an engineer
Cx: x is clever
$\exists x(Ex \& Cx), \sim Eo \models \sim Co$

(e) If everyone is here then someone who is here will not be here tomorrow.

Hx: x is here
Tx: x will be here tomorrow
Px: x is a person
$(\forall x(Px \rightarrow Hx) \rightarrow \exists x(Px \& (Hx \& \sim Tx)))$

10/10

8. (10 marks)

   Which is the following is a WFF of MPL?
   Circle each one that is a WFF of MPL.
   For each one that is not a WFF of MPL, explain why it is not a WFF.

   $(\forall xQx \& \sim\sim\sim \forall xQy)$

   not a WFF because "$\forall xQy$" can't be formed by the syntactic rules, and so can't be used to make longer WFFs

   $(\exists x(Fx \rightarrow Gx) \vee \forall y \forall x(Hy \rightarrow Ha))$

   not a WFF because "$\forall y \forall x(Hy \rightarrow Ha)$" can't be formed by the syntactic rules.

   $\sim(\exists \mathbf{x}(\mathbf{Fx} \vee \mathbf{Fb}) \& \mathbf{Fa})$

   Yes, a WFF of MPL.

   $(Fa \vee \forall x(Fx \vee Fb)$

   not a WFF because it is missing a right bracket

   $\forall y(\exists xFb \rightarrow Hy)$

   not a WFF because you can't form "$\exists xFb$" by the syntactic rules

10/10