

Exercise 3.1.1a

Explain why the rule &E for MPL is a sound rule.

In MPL, if  $(\phi \& \psi)$  is true under some interpretation then  $\phi$  and  $\psi$  are true under that interpretation too. Thus, if  $(\phi \& \psi)$  is entailed by some formula or formulas, then  $\phi$  and  $\psi$  are both entailed by those formulas too. So if in a derivation  $(\phi \& \psi)$  is entailed by its dependencies, and you write down  $\phi$  or  $\psi$  with those dependencies, then the formula you write down will be entailed by its dependencies. Hence &E for MPL is a sound rule.

Exercise 3.1.2a

Explain why for any interpretation under which "Sa" is true, " $\exists xSx$ " is true too.

Consider all interpretations under which "Sa" is true. For all such interpretations, the predicate S applies to the element a. That means for all such interpretations, there exists some element in the domain to which the predicate S applies. So for all interpretations under which "Sa" is true, " $\exists xSx$ " is true too.

Exercise 3.1.2b

Show  $(Fa \& Ga) \vdash (\exists xFx \& Ga)$

1	1. $(Fa \& Ga)$	A
1	2. Fa	1 &E
1	3. $\exists xFx$	2 $\exists$ I
1	4. Ga	1 &E
1	5. $(\exists xFx \& Ga)$	3, 4 &I

Exercise 3.1.2c

Explain why " $\exists x(\exists xSx \& Rx)$ " is not a well-formed formula of MPL.

" $\exists x(\exists xSx \& Rx)$ " is not a WFF because it cannot be formed by applying the MPL formation rules as stated in [MPL03.1]. Rule 4 there stipulates that only a variable that has not occurred before can be used to generate a quantified WFF. Hence from the expression " $(\exists xSx \& Ra)$ ", " $\exists y(\exists xSx \& Ry)$ " can be formed but not " $\exists x(\exists xSx \& Rx)$ " because "x" already occurs in " $(\exists xSx \& Ra)$ ".

Exercise 3.1.2d

State Rule  $\exists$ I without the shorthand symbolism.

If you have derived  $\phi$ , and  $\phi$  contains at least one occurrence of some constant c, then for any variable v which does not occur in  $\phi$ , you can write down " $\exists$ ", followed by v, followed by an expression formed by replacing one or more occurrences of c within  $\phi$  by v, depending on everything  $\phi$  depends on.

Exercise 3.1.3a

Explain why Rule  $\forall$ E is a sound rule.

In MPL, if  $\forall v\phi$  is true under some interpretation, then  $\phi v/c$  is true under that interpretation too. Thus if  $\forall v\phi$  is entailed by some formula or formulas, then  $\phi v/c$  is entailed by those formulas too. So if, in a derivation,  $\forall v\phi$  is entailed by its dependencies, and you write down  $\phi v/c$  with those dependencies, then  $\phi v/c$  will be entailed by its dependencies. Hence  $\forall$ E is a sound rule.