Problem Set 2

Elementary Logic

Due: 15 April 2009

Name _____

Student ID Number _____

email _____

Mark _____%

Due **15 April 2009** by **4:00PM**.

Submit your problem set to Ms. Loletta Li in Main Building 312. Make sure your problem set is timestamped. Do not submit assignments by email. Late penalty: 10% for each day late. This problem set will not be accepted after 17 April.

Answer the questions on the problem set itself. Write neatly. If the grader cannot read your handwriting, you will not receive credit.

Be sure that all pages of the assignment are securely stapled together.

Check the course bulletin board for announcements about the assignment.

Do your own work. If you copy your problem set, or permit others to copy, you may fail the course.

1. (15 marks) *True or false?*
Circle 'T' if the statement is true.
Circle 'F' if the statement is false.
φ and ψ are SL WFFs.

T	F	If φ is a tautological disjunction, then each disjunct of φ is consistent.
T	F	If φ is an inconsistent conjunction, then each conjunct of φ is inconsistent.
T	F	φ is consistent with $(\varphi \& \sim \varphi)$.
T	F	If X is an inconsistent set of MPL WFFs, then every member of X is consistent.
T	F	If X is a consistent set of MPL WFFs, then some member of X is consistent.
T	F	There is an interpretation under which "$\exists x(Hx \vee Gx)$" is false and "$\forall x(Hx \& Gx)$" is true.
T	F	The following argument can be shown to be valid in SL: "If someone plays golf, then not everyone plays golf. Everyone plays golf. So, someone does not play golf."
T	F	If φ entails ψ then ψ is consistent.
T	F	"$\exists x(Qx \leftrightarrow Px)$" is a valid MPL WFF.
T	F	"It is true that" is a truth functional connective.

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2. (16 marks)
For each of the following:
Circle "valid" if it is a valid sequent.
Circle "invalid" if it is an invalid sequent.
Otherwise, don't circle anything.

$(Q\&(P \vee (\sim P\&Q))) \models (P \rightarrow \sim Q)$	valid	invalid
$(P \rightarrow (\sim Q \rightarrow Q)) \models \sim P$	valid	invalid
$(Q\&(Q \vee R)), (P \rightarrow \sim P) \models (P \rightarrow Q)$	valid	invalid
$\sim\exists x(Px\&Qx), \sim Pa \models \sim Qa$	valid	invalid
$\exists x(Px \vee Qx), Pa \models \exists x(Pa \vee Qx)$	valid	invalid
$\forall x(Px \vee Qx), (Pa \vee Ra) \models Qa$	valid	invalid
$(\forall xPx \rightarrow \forall xQx) \models \exists x(Px \rightarrow Qx)$	valid	invalid
$Pa, \forall x(Px \rightarrow Qx) \models \sim Qa$	valid	invalid

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3. (21 marks)
Translate the following statements and arguments into MPL.
Preserve as much structure as possible.
Use the following translation scheme:

b: Bach
m: Mozart
Hx: x listens to Bach
Px: x plays the harpsichord
Cx: x composed a fugue

(a) Mozart plays the harpsichord but not everyone does.

(b) Someone who listens to Bach also composed a fugue, but someone who composed a fugue does not listen to Bach.

(c) Bach plays the harpsichord only if either everyone composed a fugue or no one did.

(d) If Mozart listens to Bach and plays the harpsichord then everyone both plays the harpsichord and listens to Bach.

(e) It is not true that whoever plays the harpsichord composed a fugue. Moreover, whoever composed a fugue plays the harpsichord. So whoever did not compose a fugue does not listen to Bach.

(f) Mozart, who composed a fugue, and Bach, who listens to Bach, both play the harpsichord.

(g) Someone composed a fugue although Bach didn't.

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4. (10 marks)

Give an MPL WFF that is logically equivalent to each of the following WFFs. Your answer must include an existential quantifier if the original WFF contains a universal quantifier, and vice versa. (MPL WFF φ is logically equivalent to MPL WFF ψ if and only if φ entails ψ, and ψ entails φ.)

(a) $\exists x(Qx \vee \sim\exists y Ry)$

(b) $\sim\sim\forall x(Px \rightarrow \sim Rx)$

(c) $\exists x(\sim Rx \& \sim Px)$

(d) $\sim\sim\forall y(Ry \& \sim Qy)$

(e) $\sim\exists x(\sim Px \& \sim Qx)$

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5. (20 marks)
Determine whether the following sequents are valid. If a sequent is valid, write "valid". If not, give an interpretation which shows that the sequent is not valid.

$\sim\exists x(Ax\&Bx), \sim Ba \models Aa$

$(\forall x(Ax \rightarrow Bx) \rightarrow \exists yCy), \exists xBx \models (\forall xBx \rightarrow \exists y(Ay\& \sim By))$

$Aa, \exists x(Ax \rightarrow Bx) \models \exists xBx$

$(\forall xAx\&\forall xBx) \models \forall x(Ax\&Bx)$

$\exists x(Ax \rightarrow Bx) \models (\sim\exists xAx \vee \exists xBx)$

$\forall xBx, \forall xAx \models \forall x(Bx \rightarrow Ax)$

$\exists x(Px \vee Qx), \exists x(Qx \vee Rx), \forall x \sim Rx \models \sim Pa$

$\forall x(Bx \vee Ax) \models \exists x(Bx\& \sim Ax)$

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6. (18 marks)
For each of the following, circle either "Yes" or "No".

Is there an interpretation under which all the following MPL WFFs are true?

$\forall x(Px \rightarrow Ax)$
$\sim\exists y(\sim Ay \vee Qy)$
$\exists y\exists x(\sim Px \vee (\sim Qx \vee Ay))$
$\sim\forall x \sim \sim(Px \rightarrow Qx)$

Yes No

Is there a consistent MPL WFF which not valid and which is true under every interpretation?

Yes No

Is there an MPL WFF which is true under all and only interpretations with exactly 3 elements in its domain?

Yes No

Is there an interpretation under which " $\forall x(Px \leftrightarrow Qx)$" is false and " $\forall x(Px \rightarrow Qx)$" is true?

Yes No

Is there a consistent set of 7 MPL WFFs such that each WFF in the set is inconsistent with "$(Aa\& \sim \exists xAx)$"?

Yes No

Is there a consistent set of 6 MPL WFFs such that each pair of WFFs in the set is consistent?

Yes No

Is there an SL WFF which contains no sentence letters other than "A" and "B", and which entails every SL conditional?

Yes No

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