(3) (8 marks) Circle your answer.

Suppose our natural deduction system is revised by adding the following rule: (NR) if you have derived $(\varphi \& \psi)$, then you can write down $(\varphi \vee \psi)$, depending on everything φ depends on.

Is the revised system sound? **YES** **NO**

Is the revised system complete? **YES** **NO**

/8

(4) (8 marks) Circle your answer.

Suppose rule $\vee$I is removed from our natural deduction system.

Is the revised system sound? **YES** **NO**

Is the revised system complete? **YES** **NO**

/8

(5) (8 marks) Circle your answer.

Suppose our natural deduction system is revised by adding the following rule: (NR1) if you have derived φ and you have derived ψ then you can write down $(\varphi \rightarrow \psi)$, depending on everything ψ depends on.

Is the revised system sound? **YES** **NO**

Is the revised system complete? **YES** **NO**

/8

(6) (6 marks)

Can the truth table method be used to help show the following?
$(A\&B) \vdash \sim(\sim A \vee \sim B)$

If yes, explain how. If no, explain why not.

We can show by truth table that '$(A\&B) \models \sim(\sim A \vee \sim B)$'.
~~Since~~ For our natural deduction system that is complete,
if '$(A\&B) \models \sim(\sim A \vee \sim B)$', then it follows that
the segment must be derivable in that system, i.e.
'$(A\&B) \vdash \sim(\sim A \vee \sim B)$'.

(An answer must mention the concept of 'completeness', /6
and its relationship with 'entailment' to score any points)

1. (30 marks)
True or false?
Circle 'T' if the statement is true.
Circle 'F' if the statement is false.
"The system" is the natural deduction system for this course.
Assume that φ and ψ are WFFs of SL.

T F If a natural deduction system is sound it is also complete.

T F The conclusion of a valid argument cannot be false.

T F If the premises of an argument are all true, then the argument is valid.

T F If φ is a contradiction, then $\varphi \vdash \psi$ is derivable in the system.

T F There is a correct derivation which uses every rule of the system.

T F The system can be used to show that $\varphi \models (\psi \rightarrow \varphi)$ is a valid sequent.

T F If rule PC is removed from the system, then the resulting system would not be complete

T F $\vdash (\psi \rightarrow (\varphi \rightarrow \varphi))$ is derivable in the system.

T F There is a natural deduction system for SL which is neither complete nor sound.

T F If $\vdash (\psi \rightarrow \varphi)$ is derivable in the system then φ is a tautology.

/30

(2) (40 marks) If it is possible, show the following using the natural deduction system for this course. If it is not possible, write "not derivable".

(a) $C, (D \rightarrow E) \vdash ((A \leftrightarrow B) \rightarrow (A \rightarrow B))$