

1. (20 marks)

True or false?

Circle 'T' if the statement is true.

Circle 'F' if the statement is false.

For this question, you should assume that φ and ψ are WFFs of SL, and that "the system" refers to our natural deduction system.

- T F If φ is not a tautology, then φ is not derivable in the system.
- ☒ T ☐ F If rule $\leftrightarrow I$ is removed, then the resulting system would not be complete.
- T ☒ ☐ F Every correct derivation in the system uses at least two different rules.
- T ☒ ☐ F In a correct derivation in the system, one can assume a formula without deriving it.
- ☒ T ☐ F The truth table method can always show whether or not an SL sequent is valid.
- T ☒ ☐ F " $(A \rightarrow \sim A)$ " is an explicit contradiction.
- ☒ T ☐ F φ might not entail ψ .
- T ☒ ☐ F Any natural deduction system that is sound is also complete.
- T ☒ ☐ F No natural deduction system that is not sound is complete.
- ☒ T ☐ F There is no longest derivation in the system.
- ☒ T ☐ F Every correct derivation in the system has at least one line.
- T ☒ ☐ F If φ entails ψ , then φ can be derived from ψ in the system.

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* The 1st statement is ambiguous, dependent upon whether it is understood as '..... in the system without any premise'. If understood this way, the answer should be true.

(i) $(A \& B) \vdash \sim \sim (A \& B)$

(i) $(\sim A \vee B) \vdash (A \rightarrow B)$

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(3) (6 marks) Circle your answer.

Suppose our natural deduction system is revised by adding the following rule:
(NR3) If you have derived ψ , then you can write down $(\varphi \rightarrow \psi)$, depending on every-
thing ψ depends on.

Is the revised system sound?

YES

NO

Is the revised system complete?

YES

NO

/6

(4) (6 marks) Circle your answer.

Suppose rule $\vee E$ is removed from our natural deduction system.

Is the revised system sound?

YES

NO

Is the revised system complete?

YES

NO

/6

(5) (6 marks) Circle your answer.

Suppose rule $\sim I$ is removed from our natural deduction system.

Is the revised system sound?

☒ YES

☐ NO

Is the revised system complete?

☒ YES

☐ NO

/6

(6) (6 marks) Circle your answer.

Suppose our natural deduction system is revised by adding the following rule:

(NR4) If you have derived $(\varphi \leftrightarrow \psi)$, then you can write down $(\psi \leftrightarrow \varphi)$, depending on everything $(\varphi \rightarrow \psi)$ depends on.

Is the revised system sound?

☒ YES

☒ NO

Is the revised system complete?

☒ YES

☐ NO

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(7) (6 marks) Circle your answer.

Suppose our natural deduction system is revised by adding the following rule:

(NR5) If you have derived $\sim(\varphi \rightarrow \psi)$, then you can write down ψ , depending on everything $\sim(\varphi \rightarrow \psi)$ depends on.

Is the revised system sound?

☒ YES

☒ NO

Is the revised system complete?

☒ YES

☐ NO

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(8) (6 marks) Circle your answer.

Suppose rule PC is removed from our natural deduction system.

Is the revised system sound?

☒ YES

☐ NO

Is the revised system complete?

☒ YES

☐ NO

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