Problem Set 2

Elementary Logic II

Due: 15 April 2009

Submit your problem set to Ms. Loletta Li in Main Building 312. Make sure your problem set is timestamped. Do not submit assignments by email. Late penalty: 10% for each day late. This problem set will not be accepted after 17 April 2009.

Answer the questions on the problem set itself. Write neatly. If the grader cannot read your handwriting, you will not receive credit.

Be sure that all pages of the assignment are securely stapled together.

Check the course bulletin board for announcements about the assignment.

Do your own work.
If you copy your problem set, or permit others to copy, you may fail the course.

Name _____

Student ID Number _____

email _____

Score: _____

Due **15 April 2009** by **4:00PM**.

(1) (18 marks)

True or false?
Circle 'T' if the statement is true.
Circle 'F' if the statement is false.
For this question, you should assume that φ is a WFF of MPL, and the derivations are in our natural deduction system for MPL.

> T   F   If rule PC is removed from our MPL natural deduction system,
>
> then the resulting system would not be complete.
>
> T   F   Every correct derivation in our MPL natural deduction system
>
> uses the rule of assumption.
>
> T   F   If an MPL sequent is not valid, then our MPL natural deduction
>
> system can be used to show that the sequent is not valid.
>
> T   F φ might be false under every interpretation.
>
> T   F   The truth table method can sometimes determine whether or not an MPL formula is valid.
>
> T   F   "$\exists x(Fx \vee Fx)$" is not a valid MPL formula.
>
> T   F   There is an MPL derivation of "$\forall x(Ax \rightarrow Ax)$" with no dependencies.
>
> T   F   Any natural deduction system that is sound is also complete.
>
> T   F   If c is a constant and v is a variable, then $\varphi v/c$ is a well-formed formula of MPL.
>
> T   F   The disjunction of φ and the negation of φ is derivable
>
> with no dependencies in our MPL natural deduction system.
>
> T   F   Some correct MPL derivations have more than 1 million lines.
>
> T   F   "$\forall x(Ax \rightarrow \exists xBx)$" is a well-formed formula of MPL.

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(2) (40 marks) For each of the following, show it using natural deduction, if it is possible. Otherwise write "Not possible".

(a) $\exists x(Gx \& Hx), \forall y(Hy \rightarrow Ry) \vdash (\exists xRx \vee \exists xPx)$

(b) $\forall x \sim Gx \vdash \sim \exists xGx$

(c) $\exists x(Ax\&Bx) \vdash (\exists xAx \rightarrow \exists xBx)$

(d) $\sim\forall x \sim Ax \vdash \exists xAx$

(e) $(\forall xAx\&\forall xBx) \vdash \forall x(Ax\&Bx)$

(f) $\forall x(Ax \rightarrow Bx), Aa \vdash (\sim Aa \lor Ba)$

(g) $(\exists xAx \rightarrow \exists yBy) \vdash \forall x(Ax \rightarrow \exists yBy)$

(h) $\vdash \exists y(\exists x Bx \rightarrow By)$

(i) $\sim\exists x(Ax\& \sim Bx) \vdash \forall x(Ax \rightarrow Bx)$

(j) $\vdash ((Pa \lor \exists x Px) \rightarrow (Pa \lor \exists y Py))$

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(3) (5 marks) Circle your answer.

Suppose the MPL natural deduction system is revised by adding the following rule:
(NR) for any variable v and constant c, if you have derived $\forall v(\varphi \lor \psi)$, then you can write down $(\forall v\varphi \lor \exists v\psi)$, depending on everything $\forall v(\varphi \lor \psi)$ depends on.

Is the revised system sound?   **YES**   **NO**

Is the revised system complete?   **YES**   **NO**

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(4) (5 marks) Circle your answer.

Suppose rule $\exists E$ is removed from the MPL natural deduction system.

Is the revised system sound?   **YES**   **NO**

Is the revised system complete?   **YES**   **NO**

/5

(5) (5 marks) Circle your answer.

Suppose the MPL natural deduction system is revised by adding the following rule:
(NR1) for any variable v and constant c, if you have derived $\forall v(\varphi \& \psi)$, then you can write down $\varphi v/c$, depending on everything $\forall v(\varphi \& \psi)$ depends on.

Is the revised system sound?   **YES**   **NO**

Is the revised system complete?   **YES**   **NO**

/5

(6) (5 marks) Circle your answer.

Suppose rule $\sim I$ is removed from the MPL natural deduction system.

Is the revised system sound?   **YES**   **NO**

Is the revised system complete?   **YES**   **NO**

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(7) (5 marks) Circle your answer.

Suppose the MPL natural deduction system is revised by adding the following rule:
(NR2) If you have derived $\exists v\varphi$, then you can write down $\forall v\varphi$, depending on everything $\exists v\varphi$ depends on.

Is the revised system sound?   **YES**   **NO**

Is the revised system complete?   **YES**   **NO**

/5

(8) (10 marks) For each of the following four attempted derivations, circle each line where a rule is misused.

1	1. $(A\&B)$	A
1	2. B	1, &E
1	3. $(B \lor C)$	2, ∨I
	4. $((A\&B) \to (B \lor C))$	1,3 →I

1	1. $(B\& \sim B)$	A
2	2. A	A
1	3. $\sim A$	2,1 ∼I
1	4. $(A \to \sim A)$	2,3 →I

1	1. $\forall x(Bx\&Ca)$	A
2	2. $(Ba\&Ca)$	1, ∀E
2	3. Ba	2 &E
1	4. $\forall xBx$	3 ∀I

1	1. $\exists x(Bx \lor \exists yCy)$	A
2	2. $(Ba \lor \exists yCy)$	A
3	3. $\sim Ba$	A
2,3	4. $\exists yCy$	2,3 ∨E
5	5. Ca	A
5	6. $(\sim Ba \to Ca)$	3,5 →I
5	7. $\exists x(\sim Bx \to Cx)$	6 ∃I
2,3	8. $\exists x(\sim Bx \to Cx)$	4,5,7 ∃E
1,3	9. $\exists x(\sim Bx \to Cx)$	1,2,8 ∃E
1	10. $(Ba \to \exists x(\sim Bx \to Cx))$	3,9 →I

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(9) (7 marks)

Translate the following argument into MPL. Be sure to write down your translation scheme. Then either show that the argument is valid using natural deduction, or show that the argument is not valid by giving an appropriate interpretation.

Babies are people. Babies are cute. No person is happy who can climb a tall tree. Cute people are happy. Therefore, babies cannot climb a tall tree.

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