

Answer Key

Problem Set 3: PHIL 1068 Elementary Logic: Due 4:00PM 21 March 2011

Student ID Number _____ Name Chan

1. (21 marks)

True or false? Circle 'T' if the statement is true. Circle 'F' if the statement is false.
For this question, you should assume that φ and ψ are WFFs of SL, and that "the system" is our SL natural deduction system.

- T. ☒ F If rule $\rightarrow I$ is removed from the system,
then the resulting system would not be sound.
- T ☒ F "If ... then ..." is a truth functional connective in English.
- T ☒ F Any natural deduction system that is complete is also sound.
- T ☒ F If a valid argument has all false premises, then its conclusion is false.
- ☒ T F Some SL sequent is not valid.
- ☒ T F The disjunction of φ and the negation of φ is derivable
with no dependencies in the system.
- ☒ T F " $\sim \sim A \vee B$ " is an expression in SL.
- ☒ T F There is a derivation in the system of
" $((A \& B) \rightarrow (\sim B \rightarrow B))$ " with no dependencies.
- T ☒ F If a natural deduction system is sound it is also complete.
- T ☒ F The conclusion of a valid argument cannot be false.
- T ☒ F If the premises of an argument are all true, then the argument is valid.
- ☒ T F If φ is a contradiction, then $\varphi \vdash \psi$ is derivable in the system.
- T ☒ F If rule PC is removed from the system, then the resulting system would not be complete.
- ☒ T F There is a natural deduction system for SL which is neither complete nor sound.

/21 1.5 marks each

2. (25 marks) Circle your answer.

Suppose rule $\vee I$ is removed from the SL natural deduction system.

Is the revised system sound? ☒ YES ☐ NO

Is the revised system complete? ☐ YES ☒ NO

Suppose rule $\&I$ is removed from the SL natural deduction system.

Is the revised system sound? ☒ YES ☐ NO

Is the revised system complete? ☐ YES ☒ NO

Suppose rule $\sim I$ is removed from the SL natural deduction system.

Is the revised system sound? ☒ YES ☐ NO

Is the revised system complete? ☒ YES ☐ NO

Suppose the SL natural deduction system is revised by adding the following rule:

(NR2) If you have derived $(\varphi \rightarrow \psi)$, and derived $(\psi \rightarrow \gamma)$, then you can write down $(\varphi \rightarrow \gamma)$, depending on everything $(\varphi \rightarrow \psi)$ depends on.

Is the revised system sound? YES NO

Is the revised system complete? YES NO

Suppose our natural deduction system is revised by adding the following rule:

(NR1) if you have derived φ and you have derived ψ then you can write down $(\varphi \rightarrow \psi)$, depending on everything ψ depends on.

Is the revised system sound? YES NO

Is the revised system complete? YES NO

2.5 marks each

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3. (4 marks)

Can the truth table method be used to help show the following?

$(A \vee B) \vdash (\sim A \rightarrow B)$

If yes, explain how. If no, explain why not.

1 mark $\rightarrow$ Yes

1 mark $\rightarrow$ The truth table method can help show $(A \vee B)$ entails $(\sim A \rightarrow B)$.

2 marks $\rightarrow$ Since our natural deduction system is complete, we know that $(A \vee B) \vdash (\sim A \rightarrow B)$ is derivable

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4. (10 marks) Translate the following statements into MPL.

Preserve as much structure as possible.

Use 'F' to translate 'is funny', 'O' to translate 'is old', and 'm' to translate 'Mary'.

(a) Something is both funny and old.

$\exists x (F_x \ \& \ O_x)$ 3

(b) Everything is old but Mary is not old.

$(\forall x O_x \ \& \ \sim O_m)$ 3

(c) If Mary is funny then something is old.

$(F_m \rightarrow \exists x O_x)$ 4 /10