Answer key

Problem Set 4

PHIL 1068 Elementary Logic

Due: **11 April 2011** by **4:00PM**

Name Chan

Student ID Number

Submit your problem set to Ms. Loletta Li in Main Building 312.
Make sure your problem set is timestamped.
Do not submit assignments by email.
Late penalty: 10% for each day late.
This problem set will not be accepted after 13 April.
Answer the questions on the problem set itself. Write neatly.
If the grader cannot read your handwriting, you will not receive credit.
Be sure that all pages of the assignment are securely stapled together.
Check the course bulletin board for announcements about the assignment.
Do your own work.
If you copy your problem set, or permit others to copy, you may fail the course.

1. (10 marks) *True or false?*
   Circle 'T' if the statement is true.
   Circle 'F' if the statement is false.
   For this question, you should assume that φ and ψ are WFFs of MPL.

   T F "$\exists x(Gx \rightarrow Gy)$" is a valid MPL WFF.
   T F If ψ is consistent, then ψ is not valid.
   T F "Richard Nixon said that" is a truth functional connective.
   T F If X is an inconsistent set of MPL WFFs, then some member of X is inconsistent.
   T F If X is a consistent set of MPL WFFs, then some member of X is consistent.
   T F If X is an inconsistent set of MPL WFFs, then every member of X is consistent.
   T F The following argument can be shown to be valid in SL: "Someone gets an A. If someone gets an A, then Nick gets an A. So, Nick gets an A."
   T F There is an interpretation under which "$\exists x(Fx \& Gx)$" is false and "$\forall x(Hx \vee Gx)$" is true.
   T F If φ entails ψ then ψ is consistent.
   T F "$\exists x(\sim Qx \rightarrow Px)$" is a valid MPL WFF.

1 mark each /10

2. (10 marks)
For each of the following:
Circle "valid" if it is a valid sequent.
Circle "invalid" if it is an invalid sequent.
Otherwise, don't circle anything.

$\sim\exists x(Px\&Qx), \sim Pa \models \sim Qa$	valid	invalid	— 2 marks
$\exists x(Px \vee Qx), Pa \models \exists x(Pa \vee Qx)$	valid	invalid	— 2 marks
$\forall x(Px \vee Qx), (Pa \vee Ra) \models Qa$	valid	invalid	1 mark each
$(\forall xPx \rightarrow \forall xQx) \models \exists x(Px \rightarrow Qx)$	valid	invalid	
$(Q\&(P \vee (\sim P\&Q))) \models (P \rightarrow \sim Q)$	valid	~~invalid~~	
$(P \rightarrow (\sim Q \rightarrow Q)) \models \sim P$	valid	~~invalid~~	
$(Q\&(Q \vee R)), (P \rightarrow \sim P) \models (P \rightarrow Q)$	valid	invalid	
$\exists x(Px\&Qx) \models (\exists xPx\&\exists yQy)$	valid	invalid	

/10

3. (15 marks)

Translate the following statements and arguments into MPL.
Preserve as much structure as possible.
Use the following translation scheme:

a: Aaron
b: Bill
Hx: x is happy
Gx: x is generous

(a) If someone is generous, then not everyone is happy.

$(\exists x\, Gx \Rightarrow \sim \forall x\, Hx)$

(b) No one is happy unless Bill is generous.

$(\sim Gb \Rightarrow \sim\exists x\, Hx)$ / $(Gb \vee \sim\exists x\, Hx)$ / $(\sim\exists x\, Hx \vee Gb)$ / $(\exists x\, Hx \rightarrow Gb)$

(c) If everyone is happy then Bill is not happy. No one is both generous and happy. But Aaron is not generous. So either every generous person is not happy or Aaron is happy.

$(\forall x\, Hx \Rightarrow \sim Hb), \sim\exists x\,(Gx \,\&\, Hx), \sim Ga \models (\forall x\,(Gx \rightarrow \sim Hx) \vee Ha)$

(d) No one generous is not happy, but someone happy is not generous.

$(\sim \exists x (Gx \& \sim Hx) \ \& \ \exists x (Hx \& \sim Gx))$

(e) Bill and Aaron are both generous if exactly one of the two is happy.

$(((Hb \vee Ha) \& \sim(Hb \& Ha)) \rightarrow (Gb \& Ga))$

3 marks each /15

4. (10 marks)

Give an MPL WFF that is logically equivalent to each of the following WFFs. Your answer must include an existential quantifier if the original WFF contains a universal quantifier, and vice versa.
(MPL WFF φ is logically equivalent to MPL WFF ψ if and only if φ entails ψ and ψ entails φ.)

(a) $\forall x(Fx\&Gx)$

$\sim \exists x \sim (Fx \& Gx)$

(b) $\sim\exists x(Fx \vee Gx)$

$\forall x \sim (Fx \vee Gx)$

5 marks each /10

5. (10 marks)

Is there an interpretation under which all the following MPL WFFs are true? If yes, then give one such interpretation. If not, explain why there is no such interpretation.

$\exists x \sim (Ax\&(Bx\&Cx))$
$\forall x(Cx \leftrightarrow Bx)$
$\exists y(Ay\& \sim By)$
$\forall x(Ax \rightarrow (\sim Cx \vee Bx))$

Yes

Domain : Living organisms

Ax : x can fly

Bx : x is a human being

Cx : x is Homo sapiens

/10

6. (10 marks)
Is there a consistent MPL WFF which is false under every interpretation? If so, give such a WFF. If not, explain why there is no such WFF.

No

An MPL WFF is consistent just in case it is true under at least one interpretation

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7. (10 marks)
Give an interpretation under which "$\exists x(Px \& Qx)$" is false and "$\forall x(Px \lor Qx)$" is true.

Domain : Girls

Px : x is female

Qx : x is male

/10

8. (15 marks) For each of the following, if it is possible, show it using our MPL natural deduction system. If it is not possible, write "not derivable".

(a) $\forall x(Ax \to Bx) \vdash {\sim}\exists x(Ax \& {\sim} Bx)$

1	1.	$\forall x(Ax \to Bx)$	A
2	2.	$\exists x(Ax \& {\sim}Bx)$	A
3	3.	$(Aa \& {\sim}Ba)$	A
1	4.	$(Aa \to Ba)$	1 $\forall$E
3	5.	Aa	3 & E
1,3	6.	Ba	4,5 $\to$E
3	7.	${\sim}Ba$	3 &E
1,3	8.	$(Ba \& {\sim}Ba)$	6,7 &I
1,3	9.	${\sim}\exists x(Ax \& {\sim}Bx)$	2,8 ~I
1,2	10.	${\sim}\exists x(Ax \& {\sim}Bx)$	2,3,9 $\exists$E
1,2	11.	$(\exists x(Ax \& {\sim}Bx) \& {\sim}\exists x(Ax \& {\sim}Bx))$	2,10 &I
1	12.	${\sim}\exists x(Ax \& {\sim}Bx)$	2,11 ~I

4 marks

(b) $\vdash \forall x(\exists y Ay \to Ax)$

Not Derivable

3 marks

(c) $\exists x(Ax \& Bx) \vdash (\exists x Ax \rightarrow \exists x Bx)$

1	1. $\exists x\ (Ax \& Bx)$	A	
2	2. $\exists x\ Ax$	A	
3	3. $(Aa \& Ba)$	A	
3	4. Ba	3 &E	
3	5. $\exists x\ Bx$	4 $\exists$I	
1	6. $\exists x\ Bx$	1,3,5 $\exists$E	4 marks
1	7. $(\exists x\ Ax \rightarrow \exists x\ Bx)$	2,6 $\rightarrow$I	

(d) $(Ab \rightarrow \forall x Bx) \vdash \forall x(Ab \rightarrow Bx)$

1	1. $(Ab \rightarrow \forall x\ Bx)$	A	
2	2. Ab	A	
1,2	3. $\forall x\ Bx$	1,2 $\rightarrow$E	
1,2	4. Bc	3 $\forall$E	
1	5. $(Ab \rightarrow Bc)$	2,4 $\rightarrow$I	4 marks
1	6. $\forall x\,(Ab \rightarrow Bx)$	5 $\forall$I	

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