Answer key

Midterm: PHIL 1068 Elementary Logic: 1 March 2011

Student ID Number _____ Name ___Chun_____

1. (15 marks)
   True or false? Circle 'T' if the statement is true. Circle 'F' if the statement is false.
   For this question, you should assume that φ is a WFF of SL.

   T **(F)** If an argument is not sound, then that argument is not valid.
   T **(F)** The conclusion of a valid argument must be true.
   T **(F)** Every sentence in English is a statement.
   T **(F)** If φ contains the symbol "$\sim$" then φ contains a two-place connective.
   T **(F)** '$(A \to B)$' entails '$(A \leftrightarrow B)$'.
   T **(F)** The consequent of '$(\sim P \to Q)$' is '$\sim P$'.
   T **(F)** If φ is a tautology, then φ is not a conjunction.
   T **(F)** If φ is consistent, then φ contains at least one connective.
   (T) F '$(A\,\&\sim A)$' entails φ.
   T **(F)** '$(\sim A \leftrightarrow A)$' is logically equivalent to '$(\sim P \vee \sim(P \to P))$'.

   1.5 mark each

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2. (5 marks)
   Write down an invalid SL sequent without using the symbol '$\sim$':

   $$A \vDash B$$

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3. *Make a truth table for each of the following WFFs of SL.* (15 marks)

   a. $(Q \leftrightarrow (Q \vee P))$

Q	P
T	T
T	F
F	T
F	F

   $(Q \leftrightarrow (Q \vee P))$
   T
   T
   F
   T

   b. $((P \leftrightarrow Q) \to (Q \vee R))$

P	Q	R
T	T	T
T	T	F
T	F	T
T	F	F
F	T	T
F	T	F
F	F	T
F	F	F

   $((P \leftrightarrow Q) \to (Q \vee R))$
   T
   T
   T
   T
   T
   T
   T
   F

   c. $((P \vee \sim \sim Q) \to \sim P)$

P	Q
T	T
T	F
F	T
F	F

   $((P \vee \sim \sim Q) \to \sim P)$
   F
   F
   T
   T

d. $\sim((P\&Q)\&(P \lor R))$

$\sim \ ((P \& Q) \& (P \lor R))$

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4. (10 marks) *Fill in the blanks with a WFF of SL to make correct truth tables.*
   Each WFF you write must be a conditional.

R	Q	P	$((R \lor Q) \to (Q \leftrightarrow \sim P))$
T	T	T	F
T	T	F	T
T	F	T	T
T	F	F	F
F	T	T	F
F	T	F	T
F	F	T	T
F	F	F	T

A	$(A \to \sim A)$
T	F
F	T

5 marks each

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5. (10 marks) Suppose that a new two-place connective '#' is added to SL.
   You are informed that "$(A \to B)$" entails "$(A\#B)$",
   and that "$(A\#B)$" is not a tautology.

   If possible, complete the following truth table.

   If it is not possible to complete the truth table, explain why.

A	B	$(A\#B)$
T	T	T
T	F	F
F	T	T
F	F	T

10 marks /10

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6. (5 marks) *Which of the following is a valid argument?*
   Circle "Yes" if it is a valid argument. Circle "No" if it is not a valid argument.

   ~~Yes~~ No   If Henry is unusual, then David is considerate.
                  David is not considerate.
                  Therefore, Henry is not unusual.

   Yes   ~~No~~   If Henry is unusual, then David is considerate.
                  Either Henry is not unusual or David is considerate.
                  Therefore, if Henry is unusual, then Rex is unique.

   Yes   ~~No~~   Hong Kong is crowded but lively.
                  San Francisco is crowded only if Hong Kong is lively.
                  Therefore, San Francisco is crowded.

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7. (10 marks)

   Translate the following statements into SL, preserving as much structure as possible.
   Be sure to write down your translation scheme.

   (a) Both you and I are happy, but neither of us is wealthy.

   Y : You are happy
   W : You are wealthy
   I : I am happy
   J : I am wealthy

   $$((Y \& W) \& (\sim I \& \sim J))$$ ~~✗~~

   (b) Provided that you have studied hard, you will pass the course.

   S : You have studied hard.
   P : You will pass the course

   $$(S \rightarrow P)$$

   (c) I think that if Hong Kong delinks from the US dollar, inflation will ensue.

   D : I think there if Hong Kong delinks from the US dollar, inflation will ensue.

   $$D$$

   (d) Unless you are inconsiderate, you should not complain.

   C : You are inconsiderate
   L : You should complain

   $$(\sim C \rightarrow \sim L) \quad / \quad (\sim L \vee C) \quad / \quad (C \vee \sim L)$$

   (e) That policy is acceptable only if the citizens agree.

   A : That policy is acceptable
   E : The citizens agree

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   $$(A \rightarrow E)$$

   2 marks each
   0.5 for translation scheme
   1.5 for translation

   OR: C : You are considerate.
   L : You should complain

   $$(C \rightarrow \sim L) \quad / \quad (\sim L \vee \sim C) / \quad (\sim C \vee \sim L)$$

8. (10 marks)
    For each of the following:
    Circle "tautology" if it is a WFF of SL that is a tautology.
    Circle "contingent" if it is a contingent WFF of SL.
    Circle "inconsistent" if it is an inconsistent WFF of SL.
    Otherwise, don't circle anything.

$$(\sim R \leftrightarrow ((R\&Q) \rightarrow (R\&P)))$$

tautology   contingent   inconsistent

$$((\sim P \vee \sim R) \rightarrow (\sim P \vee \sim Q))$$

tautology   contingent   inconsistent

$$((\sim P \rightarrow (Q \rightarrow P))\&B)$$

tautology   contingent   inconsistent

$$(\sim R\&(\sim Q \rightarrow \sim R))$$

tautology   contingent   inconsistent

$$((P \leftrightarrow \sim Q) \vee ((\sim R \leftrightarrow P)\&(R \rightarrow R)))$$

tautology   contingent   inconsistent

2 marks each /10

9. (20 marks) If it is possible, show the following using the natural deduction system for this course. If it is not possible, write "not derivable".

    (a) $\vdash (P \vee \sim P) \vee Q$

    Not   derivable

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(b) $\vdash (P \leftrightarrow P)$

(	1.	P	A
	2.	$(P \rightarrow P)$	1,1 $\rightarrow I$
	3.	$((P \rightarrow P) \& (P \rightarrow P))$	2,2 $\& I$
	4.	$(P \leftrightarrow P)$	3 $\leftrightarrow I$

(c) $\vdash ((P \rightarrow Q) \lor \sim Q))$

Not derivable

(d) $(A \leftrightarrow B) \vdash (A \leftrightarrow (A\&B))$

1	1.	$(A \leftrightarrow B)$	A
1	2.	$((A \rightarrow B) \& (B \rightarrow A))$	1. $\leftrightarrow$ E
1	3.	$(A \rightarrow B)$	2. $\&$ E
4	4.	A	A
1,4	5.	B	3,4. $\rightarrow$ E
1,4	6.	$(A \& B)$	4,5 $\& I$
1	7.	$((A \rightarrow (A \& B))$	4,6 $\rightarrow I$
8	8.	$(A \& B)$	A
1	9.	$(B \rightarrow A)$	2 $\&$ E
8	10.	B	8 $\&$ E
1,8	11.	A	9,10 $\rightarrow$ E
1	12.	$((A \& B) \rightarrow A)$	8, 11 $\rightarrow I$
1	13.	$((A \rightarrow (A \& B))$ $\&$ $((A \& B) \rightarrow A))$	7,12 $\& I$
1	14.	$(A \leftrightarrow (A \& B))$	13 $\leftrightarrow I$

(e) $(A \rightarrow B) \vdash (\sim B \rightarrow \sim A)$

1	1.	$(A \rightarrow B)$	A
2	2.	A	A
1,2	3.	B	1,2 $\rightarrow$ E
4	4.	$\sim B$	A
1,2,4	5.	$(B \& \sim B)$	3,4 $\& I$
1,4	6.	$\sim A$	2,5 $\sim I$
1	7.	$(\sim B \rightarrow \sim A)$	4,6. $\rightarrow I$

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