Problem Set 2: PHIL 1068 Elementary Logic: **Due 4:00PM 16 February 2011**

Student ID Number _____ Name _____

1. (10 marks)
   True or false? Circle 'T' if the statement is true. Circle 'F' if the statement is false. For this question, you should assume that φ is a WFF of SL.

   T F '$A \models (A\& \sim A)$' is a valid sequent.
   T F '$(A\&B)$' entails '$(B \vee \sim D)$'.
   T F φ entails '$(A \vee \sim A)$'.
   T F 'A' is a tautology.
   T F '$(P\&Q)$' is logically equivalent to '$\sim(\sim P \vee \sim Q)$'.

   /10

2. (12 marks)

   Translate the following statements into SL. Preserve as much structure as possible. Use the following translation scheme:

   P: Peter is friendly.
   Q: Quint is guilty.
   R: Rex is nice.
   S: Sarah is nice.

   (a) Peter is not friendly only if neither Rex nor Sarah is nice.

   (b) Sarah is nice, although Rex is not.

   (c) If Quint is guilty, then either both Sarah and Rex are nice or neither one is.

   (d) Whether or not Quint is guilty, Peter is friendly.

   /12

3. (5 marks)

Suppose the following two statements are **not both true***:*

If freedom is possible, then love is not possible.

Either love is not possible or hatred is possible.

Translate each of the two statements into SL, preserving as much structure as possible. Be sure to write down your translation scheme.

Is love possible?

/5

4. (15 marks)

Two of the following three statements are false and one is true:

If Lily eats then Ella's phone is broken.

Lily and Herman both smile only if Bo is not here.

If Herman does not smile, then Ella's phone is broken, but if Ella's phone is broken then Herman does not eat.

1. *Translate each of the three statements into SL, preserving as much structure as possible. Be sure to write down your translation scheme.*

2. *Which one of the three statements is true?*

3. *Is Bo here?*

/15

5. (10 marks)
For each of the following:
Circle "tautology" if it is a WFF of SL that is a tautology.
Circle "contingent" if it is a contingent WFF of SL.
Circle "inconsistent" if it is an inconsistent WFF of SL.
Otherwise, don't circle anything.

$$((\sim P \& (Q \& R)) \rightarrow (P \vee R))$$
tautology contingent inconsistent

$$(\sim P \rightarrow (Q \rightarrow P)) \& R)$$
tautology contingent inconsistent

$$((P \leftrightarrow Q) \leftrightarrow ((P \vee R) \leftrightarrow (P \& R)))$$
tautology contingent inconsistent

$$((P \rightarrow \sim Q) \& ((\sim Q \rightarrow P) \vee (P \rightarrow Q)))$$
tautology contingent inconsistent

$$(\sim R \rightarrow ((P \& Q) \rightarrow (R \vee P)))$$
tautology contingent inconsistent

$$(P \leftrightarrow (R \leftrightarrow P)) \& Q)$$
tautology contingent inconsistent

$$((P \vee \sim Q) \rightarrow \sim P)$$
tautology contingent inconsistent

$$((\sim P \rightarrow (Q \rightarrow R)) \vee ((Q \rightarrow R) \rightarrow P))$$
tautology contingent inconsistent

$$((\sim P \rightarrow \sim Q) \& ((\sim Q \leftrightarrow P) \& (R \rightarrow R)))$$
tautology contingent inconsistent

$$((P \rightarrow P) \vee (Q \rightarrow \sim Q))$$
tautology contingent inconsistent

/10

6. (8 marks) For each of the following three attempted derivations, circle each line where a rule is misused.

1	1. $(B\& \sim B)$	A
2	2. A	A
2	3. $\sim A$	2,1 $\sim$I
	4. $(A \rightarrow \sim A)$	2,3 $\rightarrow$I

1	1. A	A
	2. $(A \rightarrow A)$	1,1 $\rightarrow$I
3	3. B	A
3	4. $(A\&B)$	2,3 &I
1,3	5. $((A\&B) \vee A)$	4,1 $\vee$I

1	1. $(A \vee B)$	A
2	2. B	A
2	3. $(B \vee C)$	2, $\vee$I
2	4. $((A \vee B) \rightarrow (B \vee C))$	1,3 $\rightarrow$I

/8

7. (40 marks) For the following, if it is possible, show it using the SL natural deduction system from this course. If it is not possible, write "not derivable".

(a) $\sim \sim P \vdash P$

(b) $\vdash ((P\&Q) \rightarrow R)$

(c) $\vdash ((Q\&P) \leftrightarrow (P\&Q))$

(d) $(P \rightarrow Q) \vdash (\sim P \vee Q)$

(e) $Q, \sim P \vdash \sim(Q \rightarrow P)$

(f) $\vdash ((P \rightarrow Q) \vee (Q \rightarrow P))$

(g) $\vdash ((P \rightarrow Q) \rightarrow R) \rightarrow (P \rightarrow (Q \rightarrow R))$

(h) $\sim(\sim R \& \sim P) \vdash (R \vee P)$

(i) $(R \leftrightarrow P), \sim P, (R \vee P) \vdash (Q \rightarrow R)$

(j) $(Q \vee P) \vdash (\sim Q \rightarrow P)$

/40