Problem Set 4
PHIL 1068 Elementary Logic
Due: **11 April 2011** by **4:00PM**

Name _____

Student ID Number _____

Submit your problem set to Ms. Loletta Li in Main Building 312.

Make sure your problem set is timestamped.

Do not submit assignments by email.

Late penalty: 10% for each day late.

This problem set will not be accepted after 13 April.

Answer the questions on the problem set itself. Write neatly.

If the grader cannot read your handwriting, you will not receive credit.

Be sure that all pages of the assignment are securely stapled together.

Check the course bulletin board for announcements about the assignment.

Do your own work.

If you copy your problem set, or permit others to copy, you may fail the course.

1. (10 marks) *True or false?*
   Circle 'T' if the statement is true.
   Circle 'F' if the statement is false.
   For this question, you should assume that φ and ψ are WFFs of MPL.

   T   F   "$\exists x(Gx \rightarrow Gy)$" is a valid MPL WFF.

   T   F   If ψ is consistent, then ψ is not valid.

   T   F   "Richard Nixon said that" is a truth functional connective.

   T   F   If X is an inconsistent set of MPL WFFs, then some member of X is inconsistent.

   T   F   If X is a consistent set of MPL WFFs, then some member of X is consistent.

   T   F   If X is an inconsistent set of MPL WFFs, then every member of X is consistent.

   T   F   The following argument can be shown to be valid in SL: "Someone gets an A.
   If someone gets an A, then Nick gets an A. So, Nick gets an A."

   T   F   There is an interpretation under which "$\exists x(Fx \& Gx)$" is false
   and "$\forall x(Hx \vee Gx)$" is true.

   T   F   If φ entails ψ then ψ is consistent.

   T   F   "$\exists x(\sim Qx \rightarrow Px)$" is a valid MPL WFF.

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2. (10 marks)

For each of the following:
Circle "valid" if it is a valid sequent.
Circle "invalid" if it is an invalid sequent.
Otherwise, don't circle anything.

$$\sim\exists x(Px\&Qx), \sim Pa \models \sim Qa \quad \text{valid} \quad \text{invalid}$$
$$\exists x(Px \lor Qx), Pa \models \exists x(Pa \lor Qx) \quad \text{valid} \quad \text{invalid}$$
$$\forall x(Px \lor Qx), (Pa \lor Ra) \models Qa \quad \text{valid} \quad \text{invalid}$$
$$(\forall xPx \to \forall xQx) \models \exists x(Px \to Qx) \quad \text{valid} \quad \text{invalid}$$
$$(Q\&(P \lor (\sim P\&Q))) \models (P \to \sim Q) \quad \text{valid} \quad \text{invalid}$$
$$(P \to (\sim Q \to Q)) \models \sim P \quad \text{valid} \quad \text{invalid}$$
$$(Q\&(Q \lor R)), (P \to \sim P) \models (P \to Q) \quad \text{valid} \quad \text{invalid}$$
$$\exists x(Px\&Qx) \models (\exists xPx\&\exists yQy) \quad \text{valid} \quad \text{invalid}$$

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3. (15 marks)

Translate the following statements and arguments into MPL.
Preserve as much structure as possible.
Use the following translation scheme:

a: Aaron
b: Bill
Hx: x is happy
Gx: x is generous

(a) If someone is generous, then not everyone is happy.

(b) No one is happy unless Bill is generous.

(c) If everyone is happy then Bill is not happy. No one is both generous and happy. But Aaron is not generous. So either every generous person is not happy or Aaron is happy.

(d) No one generous is not happy, but someone happy is not generous.

(e) Bill and Aaron are both generous if exactly one of the two is happy.

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4. (10 marks)

Give an MPL WFF that is logically equivalent to each of the following WFFs. Your answer must include an existential quantifier if the original WFF contains a universal quantifier, and vice versa.

(MPL WFF φ is logically equivalent to MPL WFF ψ if and only if φ entails ψ and ψ entails φ.)

(a) $\forall x(Fx \& Gx)$

(b) $\sim\exists x(Fx \vee Gx)$

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5. (10 marks)

Is there an interpretation under which all the following MPL WFFs are true? If yes, then give one such interpretation. If not, explain why there is no such interpretation.

$\exists x \sim (Ax \& (Bx \& Cx))$
$\forall x(Cx \leftrightarrow Bx)$
$\exists y(Ay \& \sim By)$
$\forall x(Ax \rightarrow (\sim Cx \vee Bx))$

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6. (10 marks)
    Is there a consistent MPL WFF which is false under every interpretation? If so, give such a WFF. If not, explain why there is no such WFF.

7. (10 marks)
    Give an interpretation under which "$\exists x(Px \& Qx)$" is false
    and "$\forall x(Px \lor Qx)$" is true.

8. (15 marks) For each of the following, if it is possible, show it using our MPL natural deduction system. If it is not possible, write "not derivable".

    (a) $\forall x(Ax \rightarrow Bx) \vdash {\sim}\exists x(Ax \& \sim Bx)$

    (b) $\vdash \forall x(\exists y Ay \rightarrow Ax)$

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(c) $\exists x(Ax \& Bx) \vdash (\exists x Ax \rightarrow \exists x Bx)$

(d) $(Ab \rightarrow \forall x Bx) \vdash \forall x(Ab \rightarrow Bx)$

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