Problem Set 2

PHIL 2006 Logic for Philosophers 1

Due: **11 April 2011** by **4:00PM**

Name ______________________________

Student ID Number ______________________________

Submit your problem set to Ms. Loletta Li in Main Building 312.
Make sure your problem set is timestamped.
Do not submit assignments by email.
Late penalty: 10% for each day late.
This problem set will not be accepted after 13 April.
Answer the questions on the problem set itself. Write neatly.
If the grader cannot read your handwriting, you will not receive credit.
Be sure that all pages of the assignment are securely stapled together.
Check the course bulletin board for announcements about the assignment.
Do your own work.
If you copy your problem set, or permit others to copy, you may fail the course.

1. (10 marks) *True or false?*
   Circle 'T' if the statement is true.
   Circle 'F' if the statement is false.
   For this question, you should assume that φ and ψ are WFFs of MPL.

   T F "$\exists x(Gx \rightarrow Gy)$" is a valid MPL WFF.
   T F If ψ is consistent, then ψ is not valid.
   T F "Richard Nixon said that" is a truth functional connective.
   T F If X is an inconsistent set of MPL WFFs, then some member of X is inconsistent.
   T F If X is a consistent set of MPL WFFs, then some member of X is consistent.
   T F If X is an inconsistent set of MPL WFFs, then every member of X is consistent.
   T F The following argument can be shown to be valid in SL: "Someone gets an A. If someone gets an A, then Nick gets an A. So, Nick gets an A."
   T F There is an interpretation under which "$\exists x(Fx \& Gx)$" is false and "$\forall x(Hx \vee Gx)$" is true.
   T F If φ entails ψ then ψ is consistent.
   T F "$\exists x(\sim Qx \rightarrow Px)$" is a valid MPL WFF.

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2. (10 marks)

For each of the following:
Circle "valid" if it is a valid sequent.
Circle "invalid" if it is an invalid sequent.
Otherwise, don't circle anything.

$\sim\exists x(Px\&Qx), \sim Pa \models \sim Qa$	valid	invalid
$\exists x(Px \vee Qx), Pa \models \exists x(Pa \vee Qx)$	valid	invalid
$\forall x(Px \vee Qx), (Pa \vee Ra) \models Qa$	valid	invalid
$(\forall xPx \rightarrow \forall xQx) \models \exists x(Px \rightarrow Qx)$	valid	invalid
$(Q\&(P \vee (\sim P\&Q))) \models (P \rightarrow \sim Q)$	valid	invalid
$(P \rightarrow (\sim Q \rightarrow Q)) \models \sim P$	valid	invalid
$(Q\&(Q \vee R)), (P \rightarrow \sim P) \models (P \rightarrow Q)$	valid	invalid
$\exists x(Px\&Qx) \models (\exists xPx\&\exists yQy)$	valid	invalid

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3. (15 marks)

Translate the following statements and arguments into MPL.
Preserve as much structure as possible.
Use the following translation scheme:

a: Aaron
b: Bill
Hx: x is happy
Gx: x is generous

(a) If someone is generous, then not everyone is happy.

(b) No one is happy unless Bill is generous.

(c) If everyone is happy then Bill is not happy. No one is both generous and happy. But Aaron is not generous. So either every generous person is not happy or Aaron is happy.

(d) No one generous is not happy, but someone happy is not generous.

(e) Bill and Aaron are both generous if exactly one of the two is happy.

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4. (10 marks)

Give an MPL WFF that is logically equivalent to each of the following WFFs. Your answer must include an existential quantifier if the original WFF contains a universal quantifier, and vice versa.
(MPL WFF φ is logically equivalent to MPL WFF ψ if and only if φ entails ψ and ψ entails φ.)

(a) $\forall x(Fx\&Gx)$

(b) $\sim\exists x(Fx \vee Gx)$

(c) $\forall y(Fy\& \sim Fy)$

(d) $\sim\forall x(Fx \rightarrow \sim Gx)$

(e) $\exists x(\sim Fx \leftrightarrow Gx)$

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5. (10 marks)
Is there an interpretation under which all the following MPL WFFs are true? If yes, then give one such interpretation. If not, explain why there is no such interpretation.

$\exists x \sim (Ax \& (Bx \& Cx))$
$\forall x (Cx \leftrightarrow Bx)$
$\exists y (Ay \& \sim By)$
$\forall x (Ax \rightarrow (\sim Cx \vee Bx))$

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6. (10 marks)
Is there a consistent MPL WFF which is false under every interpretation? If so, give such a WFF. If not, explain why there is no such WFF.

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7. (10 marks)
A 4-interpretation is an interpretation which has more than 4 elements in its domain. Write down a consistent MPL WFF which is false under every 4-interpretation.

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8. (15 marks)

Suppose that a new one-place connective '@', and a new two-place connective '#' are added to SL. You are informed that:
'$((@A\#B) \leftrightarrow @A)$' is a tautology
'$((\sim @A\#B) \leftrightarrow \sim @A)$' is a tautology
'@A' is not contingent
'$(A \vee A)$' entails '@A'

Fill in the truth tables for '#' and '@':

A	@A
T	
F	

A	B	(A#B)
T	T	
T	F	
F	T	
F	F	

Circle 'T' if the statement is true. Circle 'F' if the statement is false:

T F '$(@A\&B)$' is contingent.
T F '$@A, (A \rightarrow B) \models B$' is a valid sequent.
T F '$(B\#A)$' is logically equivalent to '$(B\&A)$'.
T F '$(@A\#@B)$' does not entail '$(B \vee A)$'.

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9. (10 marks)
Give an interpretation under which "$\exists x(Px\&Qx)$" is false and "$\forall x(Px \vee Qx)$" is true.

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